A Short Course on Mobile Processes

80 Minutes – 20 minutes
– 80 minutes.

Prerequisites

Basic knowledge on formal systems and algebra
Intuitive Understanding on concurrent computation.

Plan of Lecture

2/12

- General Introduction
  Computation by interaction
  Context of π-calculus
- Syntax
  Syntax of Core Calculus
  Extensions and Variants.
- Structural Rules
  Definition
  Examples
  Discussion on Structural Rules
- Reduction
  Definition
  Examples
  · Scope Extrusion
  · Non-determinism
  · Basic Agents
- Dynamics of Name Passing //
- Behaviour of Mobile Processes
  Input, output and bound output.
- Labelled Transition Relation
- Strong Bisimilarity
  Definition
  Examples
  Laws of Equations
- Weak Bisimilarity
  Definition
  Examples
- Application of Bisimulations
  Multiple Name Passing
  Branching / Selection
  Recursion
  First Order Functions
  Stateful Agents
  Prelude to "Functions as Processes" //

2/13

- Embedding Calculi and Languages
- Functions as Processes
  Lazy λ-calculus
  Encoding
  Basic Syntactic Properties
  Observability and Fullabstraction
  Types for λ-like Agents (*)
- Parallel Imperative Language.
  Variables
  Control Constructs
  Procedures (*)
  Operational Correspondence
  Observability
- Turing Machine in π (*)
  Basic data structures
  Dynamics //
- Introduction to Asynchronous Calculus and Combinators
- Reduction of Syntax
  Asynchronous Message and Actor
- Basic "Asynchronous" agents
  Identity Receptors
  Link Agents and others
- Behavioral Equivalences on Asynchronous Mobile Processes
  Congruence Results
  Asynchronous Observables
  Agents and Observability (*)
- Embedding Synchrony in Asynchrony
  Encoding
  Proof Outline
  Extensions and Refinement //

(*) Optional.

2/17

- Combinators in Concurrency Setting.
  - Context
  - Atoms for Interaction.
- Seven Atoms
- Eliminating Prefix (1)
  - Inductive elimination.
- Eliminating Prefix (2)
  - Pushing out bound names
  - Some equations laws.
- Representability Theorem
  - Theorem and proof outline.
- Eliminating Replication.
  - Variants of replication (*)
  - Basic ideas of decomposition.
- Representability Theorem, stronger version.
- Further Directions.

(*) Optional.

2/17

- Introduction to Universality Theorem.
- Generalised Concurrent Combinators
  - Definition
  - Examples
- Abstract Notion of Embeddings
  - Definition
  - Basic Properties
- Examples of Embeddings (*)
  - Computability
  - Shynchrony in Asyhnchrony
  - Cases for summations.
- Universality Theorem.
  - Basic constructions
  - Proof outline
  - Discussions and Comparisons.
- Remaining Issues.

(*) optional.

π-Calculus

- Syntax (terms) $P, Q, \ldots$
- structural rules $\equiv$
- Reduction $\longrightarrow$
- labelled Transition $\xrightarrow{l}$ $\left(\xrightarrow{\tau} = \longrightarrow\right)$
- Bisimilarities $\sim$, $\approx$
- Basic constructs $a(x_1 \ldots x_n).P$ $\quad \overline{a}[v_1 \ldots v_n].P$

  $a : [(x).P_1] \,\&\, [(x).P_2]$ $\quad \overline{a} : \mathrm{inl}[(v).P]$

  $\overline{a} : \mathrm{inr}[(v).Q]$

  let $X(\tilde{x}) = P$ in Q.

LTS and Bisimulations.

- Labels.

$$l ::= \overline{a}b \mid ab \mid \overline{a}(b) \mid \underbrace{a(b)}_{\text{optional.}} \mid \tau$$

- Given $\xrightarrow{l}$,

Def $\mathcal{R} \subseteq \underline{P} \times \underline{P}$ is a strong $\boxed{\text{weak}}$ bisimulation

when $P \mathcal{R} Q$ and $P \xrightarrow{l} P'$ s.t. $FN(Q) \cap BN(l) = \phi$

we have, for some Q', $Q \xrightarrow{l} Q'$ $\boxed{Q \overset{\hat{l}}{\Longrightarrow} Q'}$ and $P' \mathcal{R} Q'$.

Similarly exchanging P and Q.

$\overline{a}(b)$

$a(c)$

$$\overset{\hat{l}}{\Longrightarrow} \overset{\text{def}}{=} \begin{cases} \xrightarrow{\tau}{}^{*} & \text{if } l = \tau \\ \xrightarrow{\tau}{}^{*} \xrightarrow{l} \xrightarrow{\tau}{}^{*} & \text{if } l \neq \tau. \end{cases}$$

Labeled Transition Relation.

[COMMON]

(IN) $a x.P \xrightarrow{ab} P[b/x]$

(OUT) $\bar{a} v.P \xrightarrow{\bar{a}v} P$

(BOUT) $P \xrightarrow{\bar{a}b} P' \Rightarrow (\nu b)P \xrightarrow{\bar{a}(b)} P'$

(PAR) $P \xrightarrow{\ell} P' \Rightarrow P|R \xrightarrow{\ell} P'|R$ $\quad BN(\ell) \cap FN(R) = \emptyset$

(RES) $P \xrightarrow{\ell} P' \Rightarrow (\nu a)P \xrightarrow{\ell} (\nu a)P'$ $\quad a \notin BN(\ell)$

(REP) $!ax.P \xrightarrow{ab} !ax.P \,|\, P\langle b/x\rangle$

(CONSERVATIVE)

(COM) $P \xrightarrow{ab} P' \quad Q \xrightarrow{\bar{a}b} Q' \Rightarrow P|Q \xrightarrow{\tau} P'|Q'$

(BCOM) $P \xrightarrow{ab} P' \quad Q \xrightarrow{\bar{a}(b)} Q' \Rightarrow P|Q \xrightarrow{\tau} (\nu b)(P'|Q')$

(α) $P \equiv_\alpha P' \quad P' \xrightarrow{\ell} Q' \quad Q' \equiv_\alpha Q \Rightarrow P \xrightarrow{\ell} Q.$

(EASY)

(COM) $ax.P \,|\, \bar{a}b.Q \xrightarrow{\tau} P\langle b/x\rangle \,|\, Q$

(COM2) $!ax.P \,|\, \bar{a}b.Q \xrightarrow{\tau} !ax.P \,|\, P\langle b/x\rangle \,|\, Q$

(STR) $P \equiv P' \quad P' \xrightarrow{\ell} Q' \quad Q' \equiv Q \Rightarrow P \xrightarrow{\ell} Q.$

Prop.

(1) $P \xrightarrow[\text{cons}]{\ell} P' \Rightarrow P \xrightarrow[\text{EASY}]{\ell} P'$

(2) $P \xrightarrow[\text{EASY}]{\ell} Q \Rightarrow P \xrightarrow[\text{cons}]{\ell} P' \equiv Q.$

(3) They give the same bisimilarities.

$\because \; P \equiv P' \wedge P \xrightarrow[\text{cons}]{\ell} Q \Rightarrow \overset{\exists Q'.}{} P' \xrightarrow[\text{cons}]{\ell} Q' \wedge Q \equiv Q'.$

[symmetric]

(BIN) $ax.P \xrightarrow{a(x)} P$

(BCOM) $P \xrightarrow{a(x)} P' \quad Q \xrightarrow{\bar{a}(x)} Q' \Rightarrow P|Q \xrightarrow{\tau} (x)(P'|Q')$

Prop

Again we have the same bisimulations

∵ Because of the side condition for bound actions.

Bisimulations for:

* $P \oplus Q \sim Q \oplus P$

$$\{(P\oplus Q,\ Q\oplus P)\} \cup \{(P|Q,\ Q|P)\} \cup \equiv$$

* $P \oplus P \approx P$

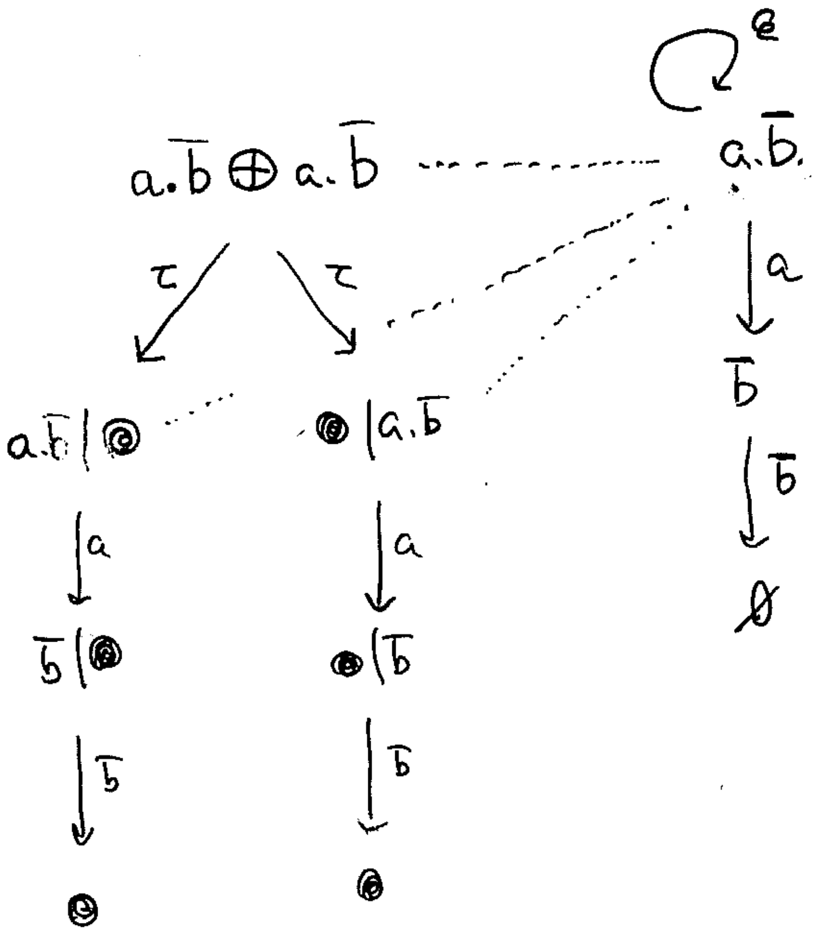
$$\{(P\oplus P,\ P)\} \cup \overset{GC}{\equiv} \cup \equiv$$

* $P \xrightarrow{\beta} Q \Rightarrow P \approx Q$

$$\{(P,\ Q) \mid P \xrightarrow{\beta} Q\} \cup \equiv$$

$C[\![(\nu c)(P_1\langle c\rangle \mid P_2\langle c\rangle)]\!]$

$$\begin{tikzcd}
 & P \arrow[dl, "\ell"'] \arrow[dr, "\beta"] & \\
Q' \not\equiv Q \arrow[dr, dashed, "\beta"'] & & Q \arrow[dl, dashed, "\ell"] \\
 & R &
\end{tikzcd}$$



$$(P \mid \bullet,\ P) \in \stackrel{GC}{\equiv}$$

Replication Law.

Prop If c occurs only as negative subjects in R_1, R_2 and P,

$$(\nu c)(!cx.P \mid R_1 \mid R_2) \stackrel{def}{\equiv} Q_1 \xrightarrow{\ell} Q_1'$$

$$\sim (\nu c)(!cx.P \mid R_1) \mid (\nu c)(!cx.P \mid R_2) \stackrel{def}{\equiv} Q_2$$

Proof: Show the following is a bisimulation:

$$\mathcal{R} = \{(Q_1', Q_2') \mid Q_1' \equiv Q_1,\ Q_2' \equiv Q_2,\ Q_1, Q_2 \text{ as above}\}$$

taking care of the invariance: c never becomes extruded nor become positive.

Counterexample.

⇒ If c occurs positively:

$$(\nu c)(!cx.\bar{e}x \mid \bar{c}v \mid cx.\bar{f}x) \xrightarrow{\tau} \xrightarrow{\bar{f}v}$$

$$\not\approx \quad (\nu c)(!cx.\bar{e}x \mid \bar{c}v) \mid$$

$$(\nu c)(!cx.\bar{e}x \mid cx.\bar{f}x)$$

⇒ If c occurs as object:

$$(\nu c)(!cx.\bar{e}x \mid \bar{c}v \mid \bar{l}c \mid ly.yx.\bar{f}x)$$

$$\longrightarrow (\nu c)(!cx.\bar{e}x \mid \bar{c}v \mid cx.\bar{f}x)$$

Pointedness.

Def a occurs active in P when:

$$P \equiv (\nu \tilde{c})(P_0 \mid R) \qquad P_0 \begin{cases} ax.P' \\ !ax.P' \end{cases} \text{Positive} \qquad \bar{a}b.P' \ \text{Negative.}$$

and $a \notin \{\tilde{c}\}$.

Def P is a-pointed when:

(1) There is a unique active occurrence of a in P

(2) No other names occur active in P

(3) $P \not\rightarrow$.

Behaviour of Agents (1)

$$a(x_1x_2).P \equiv az.zx_1.zx_2.P \xrightarrow{a(z)} \xrightarrow{zv_1} \xrightarrow{zv_2} P[v_1v_2/x_1x_2]$$

$$\bar{a}[v_1v_2].Q \equiv (\nu z)(\bar{a}z.\bar{z}v_1.\bar{z}v_2.Q) \xrightarrow{\bar{a}(z)} \xrightarrow{\bar{z}v_1} \xrightarrow{\bar{z}v_2} Q$$

Behaviour of Agents (2)

$$(\nu c_1c_2)\left(\bar{a}[c_1c_2].\left(c_1x.P \mid c_2y.Q\right)\right) \xrightarrow{\bar{a}(z)} \xrightarrow{\bar{z}(c_1)} \xrightarrow{\bar{z}(c_2)}$$

$\xrightarrow{c_1v}$ (Behaves as $P[v/x]$)

$\xrightarrow{c_2w}$ (Behaves as $Q[w/y]$)

$IM \to M \quad (I = \lambda x.x)$

$$[\![\lambda x.x]\!]c_1 = c_1(xz).\,\overline{x}z$$

$$[\![IM]\!]u = (\nu c_1 c_2)\big([\![\lambda x.x]\!]c_1 \mid \overline{c_1}[c_2 u].\,!c_2(w).\,[\![M]\!]_w\big)$$