Generalised Multiparty Session Types with Crash-Stop Failures
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Abstract

Session types enable the specification and verification of communicating systems. However, their theory often assumes that processes never fail. To address this limitation, we present a generalised multiparty session type (MPST) theory with crash-stop failures, where processes can crash arbitrarily.

Our new theory validates more protocols and processes w.r.t. previous work. We apply minimal syntactic changes to standard session π-calculus and types: we model crashes and their handling semantically, with a generalised MPST typing system parametric on a behavioural safety property. We cover the spectrum between fully reliable and fully unreliable sessions, via optional reliability assumptions, and prove type safety and protocol conformance in the presence of crash-stop failures.

Introducing crash-stop failures has non-trivial consequences: writing correct processes that handle all crash scenarios can be difficult. Yet, our generalised MPST theory allows us to tame this complexity, via model checking, to validate whether a multiparty session satisfies desired behavioural properties, e.g. deadlock-freedom or liveness, even in presence of crashes. We implement our approach using the mCRL2 model checker, and evaluate it with examples extended from the literature.

1 Introduction

Multiparty session types (MPST) [20] provide a typing discipline for message-passing processes. The theory ensures well-typed processes enjoy desirable properties, a.k.a. the Session Theorems: type safety (processes communicate without errors), protocol conformance (a.k.a. session fidelity, processes behave according to their types), deadlock-freedom (processes do not get stuck), and liveness (input/output actions eventually succeed). Researchers devote significant effort into integrating session types in programming languages and tools [16].

A common assumption in session type theory is that everything is reliable and there are no failures, which is often unrealistic in real-world systems. So, we pose a question: how can we better model systems with failures, and make session types less idealistic?
In this paper, we take steps towards bridging the gap between theory and practice with a new generalised multiparty session type theory that models failures with crash-stop semantics [4, §2.2]: processes may crash, and crashed processes stop interacting with the world. This model is standard in distributed systems, and is used in related work on session types with error-handling capabilities [33,34]. However, unlike previous work, we allow any process to crash arbitrarily, and support optional assumptions on non-crashing processes.

In our new theory, we add crashing and crash handling semantics to processes and session types. With minimal changes to the standard surface syntax, we model a variety of subtle, complex behaviours arising from unreliable communicating processes. An active process $P$ may crash arbitrarily, and a process $Q$ interacting with $P$ might need to be prepared to handle possible crashes. Messages sent from $Q$ to a crashed $P$ are lost – but if $Q$ tries to receive from $P$, then $Q$ can detect that $P$ has crashed, and take a crash handling branch.

Meanwhile, another process $R$ may (or may not) have detected $P$’s crash, and may be handling it – and in either case, any interaction between $Q$ and $R$ should remain correct.

Our MPST theory is generalised in two aspects: (1) we introduce optional reliability assumptions, so we can model a mixture of reliable and unreliable communicating peers; and (2) our type system is parametric on a type-level behavioural property $\varphi$ which can be instantiated as safety, deadlock freedom, liveness, etc. (in the style of [29]), while accounting for potential crashes. We prove session fidelity, showing how type-level properties transfer to well-typed processes; we also prove that our new theory satisfies other Session Theorems of MPST, while (unlike previous work) being resilient to arbitrary crash-stop failures.

With optional reliability assumptions, one may declare that some peers will never crash for the duration of the protocol. Such optional assumptions allow for simplifying protocols and programs: if a peer is assumed reliable, the other peers can interact with it without needing to handle its crashes. By making such assumptions explicit and customisable, our theory supports a spectrum of scenarios ranging from only having sessions with reliable peers (thus subsuming classic MPST works [20,29]), to having no reliable peers at all.

As in the real world, a system with crash-stop failures can have subtle complex behaviours; hence, writing protocols and processes where all possible crash scenarios are correctly handled can be hard. This highlights a further benefit of our generalised theory: we formalise our behavioural properties as modal $\mu$-calculus formulæ, and verify them with a model checker.

To show the feasibility of our approach, we present an accompanying tool, utilising the mCRL2 model checker [3], for verifying session properties under optional reliability assumptions.

**Overview.** Session typing systems assign session types (a.k.a. local types) to communication channels, used by processes to send and receive messages. In essence, a session type describes a protocol: how a role is expected to interact with other roles in a multiparty session. The type system checks whether a process implements desired protocols.

As an example, consider a simple Domain Name System (DNS) scenario: a client $p$ queries a server $q$ for an IP address of a host name. With classic session types (without crashes), we use the type $T_p = q@\text{req}.q@\text{res}$ to represent the client $p$’s behaviour: first sending ($\oplus$) a request message to server $q$, and then receiving ($\&$) a response from $q$. The server implements a dual type $T_q = p@\text{req}.p@\text{res}$, who receives a request from client $p$, and then sends a response to $p$. We can write a process $Q = s[q|p]@\text{req}.s[q|p]@\text{res}.0$ for the server. Using $T_q$, we type-check the channel (a.k.a. session endpoint) $s[q]$, where $Q$ plays the role $q$ on session $s$. Here, $Q$ type-checks – it uses channel $s[q]$ correctly, according to type $T_q$.

In this work, we augment the classic session types theory by introducing process failures with crash-stop semantics [4, §2.2]. We adopt the following failure model: (1) processes have
crash-stop failures, i.e. they may crash and do not recover; (2) communication channels deliver messages in order, without losses (unless the recipient has crashed); (3) each process has a failure detector \[10\], so a process trying to receive from a crashed peer accurately detects the crash. The combination of (1), (2), and (3) is called the crash-fail model in \[4, \S 2.6.2\].

We now revise our DNS example in the presence of failures. Let us assume that the server \( q \) may crash, whereas the client \( p \) remains reliable. The client \( p \) may now send its request to a new failover server \( r \) (assumed reliable for simplicity). We represent this scenario by a type for the new failover server \( T'_r \), and a new branch in \( T'_p \) for handling \( q \)'s crash:

\[
\begin{align*}
T'_p &= q \oplus \text{req, } q \& \{ \text{res} \} \\
T'_q &= p \& \text{req, } p \& \text{res} \\
T'_r &= q \& \text{crash, } p \& \text{req, } p \& \text{res}
\end{align*}
\]

Here, \( T'_p \) states that client \( p \) first sends a message to the unreliable server \( q \); then, \( p \) expects a response from \( q \). If \( q \) crashes, the client \( p \) detects the crash and handles it (via the new crash handling branch) by requesting from the failover server \( r \). Meanwhile, \( r \) also detects whether \( q \) has crashed. If so, \( r \) activates its crash handling branch and handles \( p \)'s request.

In our model, crash detection and handling is done on the receiving side, e.g. \( T'_p \) detects whether \( q \) has crashed when waiting for a response, while \( T'_q \) monitors whether \( q \) has crashed. Handling crashes when receiving messages from a reliable role is unnecessary, e.g. the server \( q \) does not need crash handling when it receives from the (reliable) client \( p \); similarly, the (reliable) roles \( p \) and \( r \) interact without crash handling. This failure model is reflected in the semantics of both processes and session types in our work. Unlike classic MPST works, we allow processes to crash arbitrarily while attempting inputs or outputs (\$2\). When a process crashes, the channel endpoints held by the process also crash, and are assigned the new type \( \text{stop} \) (\$3). E.g. when the server process \( Q \) crashes, the endpoint \( s[q] \) held by \( Q \) becomes a crashed endpoint \( s[q] \& \text{stop} \); accordingly, the server type \( T'_q \) advances to \( \text{stop} \) to reflect the crash.

To ensure that communicating processes are type-safe even in the presence of crashes, we require their session types to satisfy a safety property accounting for possible crashes (Def. 8), which can be refined, e.g. as deadlock-freedom or liveness (Def. 18). We prove subject reduction, session fidelity, and various process properties (deadlock-freedom, liveness, etc.) even in the presence of crashes and optional reliability assumptions (Thms. 11, 15, 20).

Despite minimal changes to the surface syntax of session types and processes, the semantics surrounding crashes introduce subtle behaviours and increase complexity. Taking the DNS examples above, we compare the sizes of their (labelled) transition systems in Fig. 1 (based on Def. 7): the original system (left, two roles \( p \) and \( q \), no crashes) has 10 states and 15 transitions; and the revised system (right, \( q \) may crash, with a new role \( r \)) has 101 states and 427 transitions. We discuss another, more complex example in \$4\). Checking whether a given combination of session types with possible crashes is safe, deadlock-free, or live, can be challenging due to non-trivial behaviours and increased model size arising from crashes and crash handling. To tackle this, we show how to automatically verify such type-level properties.
by representing them as modal μ-calculus formulae via the mCRL2 model checker [3].

Contributions and Structure. In §2 we introduce a multiparty session π-calculus (with minimal changes to the standard syntax) giving crash and crash handling semantics modelling crash-stop failures. In §3 we present multiparty session types with crashes: they describe how communication channels should be used to send/receive messages, and handle crashes. We formalise the semantics of collections of local types under optional reliability assumptions; we introduce a type system, and prove the Session Theorems: type safety, protocol conformance, and process properties (deadlock-freedom, termination, liveness, etc.) in Thms. 11, 15, and 20. In §4 we show how model checking can be incorporated to verify our behavioural properties, by expressing them as modal μ-calculus formulae. We discuss related work and conclude in §5. The appendices include additional examples, definitions, proofs of main theorems, and more details about the tool implementing our theory using the mCRL2 model checker.

## 2 Multiparty Session Calculus with Crash-Stop Semantics

In this section, we formalise the syntax and operational semantics of our multiparty session π-calculus, where a process can fail arbitrarily, and crashes can be detected and handled by receiving processes. For clarity of presentation, we formalise a synchronous semantics.

**Syntax of Processes.** Our multiparty session π-calculus models processes that interact via multiparty channels, and may arbitrarily crash. For simplicity of presentation, our calculus is streamlined to focus on communication; standard extensions, e.g., with expressions and “if . . . then . . . else” statements, are routine and orthogonal to our formulation.

▶ **Definition 1** (Syntax of Multiparty Session π-Calculus). Let p,q, . . . denote roles belonging to a set Π; let s, s′, . . . denote sessions; let x, y, . . . denote variables; let m,m′, . . . denote message labels; let X,Y, . . . denote process variables. The multiparty session π-calculus syntax is:

- \( c := x \mid s[p] \) (variable or channel for session s with role p)
- \( d := v \mid c \) (basic value, variable, or channel with role)
- \( w := v \mid s[p] \) (basic value or channel with role)
- \( P, Q := 0 \mid (νs)P \mid P | Q \) (inaction, restriction, parallel composition)
- \( c[q] ⊕ m(d), P \) (where m ̸= crash) (selection towards role q)
- \( c[q] & \{m_i(x_i).P_i\}_{i∈I} \) (branching from role q with an index set I ̸= ∅)
- \( \text{def } D \text{ in } P \mid X(d) \) (process definition, process call)
- \( \text{err } \mid s[p] \perp d \) (error, crashed channel endpoint)
- \( D := X(\overline{d}) ) \) (declaration of process variable X)

We write \( Π_i ∈ Π_i \) for the parallel composition of processes \( P_i \). Restriction, branching, and process definitions and declarations act as binders, as expected; \( ℓv(P) \) is the set of free channels with roles in \( P \) (including \( s[p] \) in \( s[p] \perp d \)), and \( ℓfv(P) \) is the set of free variables in \( P \). Noticeable changes w.r.t. standard session calculi are highlighted.

Our calculus (Def. 1) includes basic values \( v \) (e.g. unit ()), integers, strings), channels with roles (a.k.a. session endpoints) \( s[p] \), session scope restriction \((νs)P\), inaction \( 0 \), parallel composition \( P | Q \), process definition \( \text{def } D \text{ in } P \), process call \( X(d) \), and error \( \text{err} \). Selection (a.k.a. internal choice) \( c[q] ⊕ m(d).P \) sends a message \( m \) with payload \( d \) to role \( q \) via endpoint \( c \), where \( c \) may be a variable or channel with role, while \( d \) may also be a basic value. Branching (a.k.a. external choice) \( c[q] & \{m_i(x_i).P_i\}_{i∈I} \) expects to receive a message \( m_i \) (for some \( i ∈ I \)) from role \( q \) via endpoint \( c \), and then continues as \( P_i \). Importantly, a process implements crash
where (i.e. The reduction ▶ communicate on a session
P from a crashed endpoint, then the process detects the crash and follows its crash handling
message is lost; if the message payload is a session endpoint

\[ s[p]|q \& \{m(x_i).P_i\}_{i \in I} \mid s[q]|p \oplus m(w).Q \rightarrow P_k\{w/x_k\} \mid Q \quad \text{if } k \in I \]

\[ s[p]|q \& \{m(x_i).P_i\}_{i \in I} \mid s[q]|p \oplus m(w).Q \rightarrow \text{err} \quad \text{if } \forall i \in I : m_i \neq m \]

\[ \text{def } X(x_1, \ldots, x_n) = P \in (X(w_1, \ldots, w_n) \mid Q) \rightarrow \text{def } X(x_1, \ldots, x_n) = P \in (P\{w_1/x_1\} \cdots \{w_n/x_n\} \mid Q) \]

\[ P \rightarrow P' \text{ implies } C[P] \rightarrow C[P'] \]

\[ P' \equiv P \text{ and } P \rightarrow Q \text{ and } Q \equiv Q' \text{ implies } P' \rightarrow Q' \]

\[ P = s[p]|q \oplus m(w).P' \rightarrow \Pi_{j \in J}s_j|p_j|_{j \in J} \text{ where } \{s_j|p_j|\}_{j \in J} = \text{fe}(P) \]

\[ P = s[p]|q \& \{m(x_i).P_i\}_{i \in I} \rightarrow \Pi_{j \in J}s_j|p_j|_{j \in J} \text{ where } \{s_j|p_j|\}_{j \in J} = \text{fe}(P') \]

\[ s[p]|\pi|s[q]|p \oplus m(w).Q' \rightarrow s[p]|\pi|s[q]|p \oplus m(x)|Q' \]

\[ s[p]|q \& \{m(x_i).P_i, \text{crash}.P'\}_{i \in I} \mid s[q]|p \rightarrow P' \mid s[q]|p \]

\[ \text{Definition 2. } (\nu s) (s[p]|s'[\pi] \cdots s[p_i]|s'_{\pi_i}) \equiv 0 \quad \text{[\text{C-CrashEnd}] } \]

\[ \text{Figure 2} \text{ Semantics of our session } \pi\text{-calculus. Rule } \text{[\text{R-eq}]} \text{ uses the congruence } \equiv \text{ defined in } \S \text{ A.} \]

Operational Semantics. We give the operational semantics of our session \( \pi \)-calculus in Def. 2, using a standard structural congruence extended with a new crash elimination rule
which garbage-collects sessions where all endpoints are crashed: (full congruence rules in § A)

\[ (\nu s) (s[p]|s'[\pi] \cdots s[p_i]|s'_{\pi_i}) \equiv 0 \quad \text{[\text{C-CrashEnd}] } \]

\[ A \text{ reduction context } C \text{ is defined as: } C ::= C \mid P \mid (\nu s) C \mid \text{def } D \text{ in } C \mid [] \]

\[ \text{The reduction } \rightarrow \text{ is defined in Fig. 2; we write } \rightarrow^*/\rightarrow^* \text{ for its transitive/reflexive-transitive closure. We write } P \not\rightarrow P' \text{ iff } \not\exists P'' \text{ such that } P \rightarrow P' \text{ is derivable without rules } [\text{R-f-g}] \text{ and } [\text{R-f-k}] \]

\[ (i.e. \text{ P is stuck, unless a crash occurs). We say P has an error iff } \exists C \text{ with } P = C[\text{err}] \].

Part of our operational semantics rules in Fig. 2 are standard. Rule [\text{R-f-g}] describes a communication on session \( s \) between receiver \( p \) and sender \( q \), if the sent message \( m_k \) can be handled by the receiver \( (k \in I) \); otherwise, a message label mismatch causes an error via rule [\text{R-Err}]. Rule [\text{R-x}] expands process definitions when called. Rules [\text{R-Ctx}] and [\text{R-eq}] allow processes to reduce under reduction contexts and modulo structural congruence.

The remaining rules in Fig. 2 (highlighted) are novel: they model crashes, and crash handling.
Rules [\text{R-f-b}] and [\text{R-f-k}] state that a process \( P \) may crash while attempting any selection or branching operation, respectively; when \( P \) crashes, it reduces to a parallel composition where all the channel endpoints held by \( P \) are crashed. The lost message rules [\text{R-f-b}] and [\text{R-f-n}] state that if a process sends a message to a crashed endpoint, then the message is lost; if the message payload is a session endpoint \( s'[x] \), then it becomes crashed.
Finally, the crash handling rule [\text{R-m}] states that if a process attempts to receive a message from a crashed endpoint, then the process detects the crash and follows its crash handling branch \( P' \). We now show an example of rule [\text{R-f-b}]; more examples can be found in § C.

\[ \text{Example 3. Processes } P = s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \text{ and } Q = s[q]|p \& m'(x).x|p \oplus m(42) \text{ communicate on a session } s; P \text{ uses } s[p] \text{ to send } s[x] \text{ to role } q; Q \text{ uses } s[q] \text{ to receive it, then sends a message to role } p \text{ via } s[x]. \text{ Suppose that } P \text{ crashes before sending: this gives rise to} \]

\[ s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \rightarrow s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \]

\[ s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \rightarrow s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \]

\[ s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \rightarrow s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \]

\[ s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \rightarrow s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \]

\[ s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \rightarrow s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \]

\[ s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \rightarrow s[p]|q \oplus m'(s[x]).s[p]|r \& m(x) \]
the reduction (by rule \([R^+]\)) \((\nu s) (P \mid Q) \rightarrow (\nu s[p] \mid s[p] \mid Q)\). Observe that \(s[p]\) and \(s[r]\), which were held by \(P\), are now crashed.

3 Multiparty Session Types with Crashes

In this section, we present a generalised type system for our multiparty session \(\pi\)-calculus (introduced in Def. 1). As in standard MPST, we assign session types to channel endpoints; we show the syntax of our types in §3.1, where our key additions are crash handling branches, and a new type \(\text{stop}\) for crashed endpoints. In §3.2, we give a labelled transition system (LTS) semantics to typing contexts, to represent the behaviour of a collection of types.

Unlike classic MPST, our type system is generalised in the style of [29], hence it has no global types; rather, it uses a safety property formalising the minimum requirement for a typing context to ensure subject reduction (and thus, type safety). In this paper, such a safety property is defined in §3.3: unlike previous work, the property accounts for potential crashes, and supports explicit (and optional) reliability assumptions. We show typing rules in §3.4, and the main properties of the typing system: subject reduction (Thm. 11) and session fidelity (Thm. 15) in §3.5. Finally, we demonstrate how we can infer runtime process properties from typing contexts in §3.6.

3.1 Types

A session type describes how a process is expected to use a communication channel to send/receive messages to/from other roles involved in a multiparty session. We formalise the syntax of session types in Def. 4, where we add the \(\text{stop}\) type to their standard syntax [29].

**Definition 4 (Types).** Our types include both basic types and session types:

\[
\begin{align*}
B & ::= \text{int} \mid \text{bool} \mid \text{real} \mid \text{unit} \mid \ldots & \text{(basic types)} \\
S & ::= B \mid T & \text{(type or session)} \\
T & ::= p \& \{m_i(S_i).T_i\}_{i \in I} \mid p \oplus \{m_i(S_i).T_i\}_{i \in I} & \text{(external or internal choice, with } I \neq \emptyset) \\
& \mid \mu t.T & \text{(recursion, type variable, or termination)} \\
U & ::= T \mid \text{stop} & \text{(session type or crash type)}
\end{align*}
\]

In internal and external choices, the index set \(I\) must be non-empty, and labels \(m_i\) must be pairwise distinct. Types are always closed (i.e. each recursion variable \(t\) is bound under a \(\mu t\ldots\)) and recursion variables are guarded, i.e. they can only appear under an internal/external choice (e.g. \(\mu t.\mu t'.t\) is not a valid type). For brevity, we may omit the payload type unit and the trailing end: e.g. \(p \oplus m_1.r\&m_2\) is shorthand for \(p \oplus m_1(\text{unit}).r\&m_2(\text{unit}).\text{end}\).

The internal choice (selection) type \(p \& \{m_i(S_i).T_i\}_{i \in I}\) denotes sending a message \(m_i\) (by picking some \(i \in I\)) with a payload of type \(S_i\) to role \(p\), and then continue the protocol as \(T_i\). Dually, the external choice (branching) type \(p \oplus \{m_i(S_i).T_i\}_{i \in I}\) denotes receiving a message \(m_i\) (for any \(i \in I\)) with a payload of type \(S_i\) from role \(p\), and then continue as \(T_i\). The type end indicates that a session endpoint should not be used for further communications.

Crashes and Crash Detection. The key novelty of Def. 4 is the new type \(\text{stop}\) describing a crashed session endpoint. Similarly to Def. 1, we also introduce a distinguished message label crash for crash handling in external choices. For example, recall the types in §1:

- the type \(q \& \{\text{res.} T, \text{crash.} T'\}\) means that we expect a response message from role \(q\), but if we detect that \(q\) has crashed, then the protocol continues along the handling branch \(T'\);
- the type \(q \& \text{crash.} T\) denotes a “pure” crash recovery behaviour: we are not communicating with \(q\), but the recovery protocol \(T\) is activated whenever we detect that \(q\) has crashed.
The context composition "pure" crash recovery protocol (as outlined above), hence we do not allow the supertype to have more input branches. This way, a "pure" crash recovery type can only be implemented by a "pure" crash recovery process (with a singleton crash message label cannot appear in internal choice types).

Session Subtyping. We use a subtyping relation \( \preceq \) that is mostly standard: a subtype can have wider internal choices and narrower external choices w.r.t. a supertype. To correctly support crash handling, we apply two changes: (1) we add the relation stop \( \preceq \) stop, and (2) we treat external choices with a singleton crash branch in a special way: they represent a "pure" crash recovery protocol (as outlined above), hence we do not allow the supertype to have more input branches. This way, a "pure" crash recovery type can only be implemented by a "pure" crash recovery process (with a singleton crash detection branch); such processes are treated specially by the properties in §3.6. For the complete definition of \( \preceq \), see §B.

3.2 Typing Contexts and their Semantics

Before introducing the typing rules for our calculus (in §3.4), we first formalise typing contexts (Def. 5) and their semantics (Def. 7).

Definition 5 (Typing Contexts). \( \Theta \) denotes a partial mapping from process variables to \( n \)-tuples of types, and \( \Gamma \) denotes a partial mapping from channels to types. Their syntax is:

\[
\Theta := \emptyset \mid \Theta, X:S_1, \ldots, S_n \\
\Gamma := \emptyset \mid \Gamma, x:S \mid \Gamma, \text{s[p]}:U
\]

The context composition \( \Gamma_1, \Gamma_2 \) is defined iff dom(\( \Gamma_1 \)) \( \cap \) dom(\( \Gamma_2 \)) = \( \emptyset \). We write \( s \not\in \Gamma \) iff \( \forall p : \text{s[p]} \not\in \text{dom}(\Gamma) \) (i.e. session \( s \) does not occur in \( \Gamma \)). We write \( \Gamma \preceq \Gamma' \) iff \( \text{dom}(\Gamma) = \text{dom}(\Gamma') \) and \( \forall c \in \text{dom}(\Gamma) : \Gamma(c) \leq \Gamma'(c) \).

Unlike typical session typing systems, our Def. 5 allows a session endpoint \( \text{s[p]} \) to have either a session type \( T \), or the crash type stop. We equip our typing contexts with a labelled transition system (LTS) semantics (in Def. 7) using the labels in Def. 6.

Definition 6 (Transition Labels). Let \( \alpha \) denote a transition label having the form:

\[
\alpha := \text{s[p]:q&m(S)} \quad \text{(in session s, p receives message m(S) from q; we omit S if S = unit)} \\
\text{s[p]:q&m(S)} \quad \text{(in session s, p sends message m(S) to q; we omit S if S = unit)} \\
\text{s[p]} \quad \text{(in session s, message m is transmitted from p to q)} \\
\text{stop} \quad \text{(in session s, p crashes)} \\
\text{stop} \quad \text{(in session s, p has detected that q has crashed)} \\
\text{stop} \quad \text{(in session s, p has stopped due to a crash)}
\]
Definition 7 (Typing Context Semantics). The typing context transition $\Rightarrow$ is defined in Fig. 3. We write $\Gamma \Rightarrow s$ iff $\Gamma \Rightarrow s \Gamma'$ for some $\Gamma'$. We define the two reductions $\rightarrow$ and $\rightarrow \tau_{\setminus s, R}$ (where $s$ is a session, and $\mathcal{R}$ is a set of roles) as follows:

- $\Gamma \rightarrow \Gamma'$ holds iff $\Gamma \xrightarrow{\text{s[p][q]}} \Gamma'$ or $\Gamma \xrightarrow{\text{s[q]}} \Gamma'$ (for some $s, p, q, m$). This means that $\Gamma$ can advance via message transmission or crash detection, but it cannot advance by crashing one of its entries. We write $\Gamma \rightarrow s$ iff $\Gamma \rightarrow \Gamma'$ for some $\Gamma'$, and $\Gamma \rightarrow \tau$ for its negation (i.e. there is no $\Gamma'$ such that $\Gamma \rightarrow \Gamma'$), and $\rightarrow \tau$ for the reflexive and transitive closure of $\rightarrow$.

- $\Gamma \rightarrow \tau_{\setminus s, \mathcal{R}} \Gamma'$ holds iff $\Gamma \xrightarrow{\text{\phi}} \Gamma'$ with $\alpha \in \{s[q][\mathcal{R}]m, s[q] \odot r, s[p]s, p, q, r \in \mathcal{R}, p \notin \mathcal{R}\}$. This means that $\Gamma$ can advance via message transmission or crash detection on session $s$, involving any roles $q$ and $r$. (Recall that $\mathcal{R}$ is the set of all roles.) Moreover, $\Gamma$ can advance by crashing one of its entries $s[p]$ unless $p \in \mathcal{R}$, which means that $\Gamma$ is assumed to be reliable. We write $\Gamma \rightarrow \tau_{\setminus s, \mathcal{R}} \Gamma'$ iff $\Gamma \rightarrow \tau_{\setminus s, \mathcal{R}} \Gamma'$ for some $\Gamma'$, and $\Gamma \rightarrow \tau_{\setminus s, \mathcal{R}} \Gamma'$ for its negation, and $\rightarrow \tau_{\setminus s, \mathcal{R}}$ as the reflexive and transitive closure of $\rightarrow \tau_{\setminus s, \mathcal{R}}$. We write $\Gamma \rightarrow \tau \Gamma'$ iff $\Gamma \rightarrow \tau_{\setminus s, \mathcal{R}} \Gamma'$ for some $s$ (i.e. $\Gamma$ may advance by crashing any role on any session).

Def. 7 subsumes the standard typing context reductions [29, Def. 2.8]. Rule $[\Gamma \& \phi]$ (resp. $[\Gamma \& \phi \& \Gamma]$) says that an entry can perform an output (resp. input) transition. Rule $[\Gamma \& \phi \& \Gamma]$ synchronises matching input/output transitions, provided that the payloads are compatible by subtyping; as a result, the context advances via a message transmission label $s[p][\mathcal{R}]m$. Other standard rules are $[\Gamma \& \phi]$ for recursion, and $[\Gamma]$ and $[\Gamma \& \phi]$ for reductions in a larger context.

The key innovations are the (highlighted) rules modelling crashes and crash detection. By rule $[\Gamma \& \phi \& \Gamma]$, an entry can crash and become stop at any time (unless it is already ended or stopped); then, by rule $[\Gamma \& \phi \& \Gamma]$, it keeps signalling that it is crashed, with label $s[p]\text{stop}$. Rule $[\Gamma \& \phi]$ models crash detection and handling: if $s[p]$ signals that it has crashed and stopped, another entry $s[q]$ can then take its crash handling branch (part of an external choice from $p$). This corresponds to the process reduction rule $[\Gamma \& \phi]$ for crash detection.

Finally, rule $[\Gamma \& \phi \& \Gamma]$ models the case where the entry $s[p]$ is sending a message $m(S)$ to a crashed $s[q]$: this yields a transmission label $s[p][\mathcal{R}]m$, and $p$ continues – although the sent message is not actually received by crashed $q$. This corresponds to the process reduction rule $[\Gamma \& \phi \& \Gamma]$ where a process sends a message to a crashed endpoint, and cannot detect its crash.

### 3.3 Typing Context Safety

To ensure type safety (Cor. 12), i.e. well-typed processes do not result in errors, we define a safety property $\varphi(\cdot)$ (Def. 8) as a predicate on typing contexts $\Gamma$. The safety property $\varphi$ is the key feature of generalised MPST systems [29, Def. 4.1]; in this work, we extend its definition in two crucial ways: (1) we support crashes and crash detection, and (2) we make the property parametric upon a (possibly empty) set of reliable roles $\mathcal{R}$, thus introducing optional reliability assumptions about roles in a session that never fail.

Definition 8 (Typing Context Safety). Given a set of reliable roles $\mathcal{R}$ and a session $s$, we say that $\varphi$ is an $(s; \mathcal{R})$-safety property of typing contexts iff, whenever $\varphi(\Gamma)$, we have:

- $[\Gamma \& \phi \& \Gamma] \quad \Gamma \xrightarrow{\text{s[p][q]m(S)}}$ and $\Gamma \xrightarrow{\text{s[q]p\&m(S)}} \Gamma \Rightarrow s[p][q]m \Rightarrow \Gamma$ implies $\Gamma \xrightarrow{\text{s[p][q]m}}$;
- $[\Gamma \& \phi \& \Gamma] \quad \Gamma \xrightarrow{\text{s[p]\text{stop}}} \Gamma \Rightarrow s[p]\text{stop} \Rightarrow \Gamma \Rightarrow s[p]p\&m(S) \Rightarrow \Gamma \Rightarrow s[p]\text{stop} \Rightarrow \Gamma$ implies $\Gamma \xrightarrow{\text{s[p]\&p}}$;
- $[\Gamma \& \phi \& \Gamma] \quad \Gamma \Rightarrow \tau_{\setminus s, \mathcal{R}} \Gamma' \Rightarrow \varphi(\Gamma')$.

We say $\Gamma$ is $(s; \mathcal{R})$-safe, written safe$(s; \mathcal{R}, \Gamma)$, if $\varphi(\Gamma)$ holds for some $(s; \mathcal{R})$-safety property $\varphi$. We say $\Gamma$ is safe, written safe$(\Gamma)$, if $\varphi(\Gamma)$ holds for some property $\varphi$ which is an $(s; \emptyset)$-safety property for all sessions $s$ occurring in dom($\Gamma$).
Figure 4 Typing rules for processes; \( \varphi \) in [\( T \cdot \cdot \cdot \)] is an \( (s; \mathcal{R}) \)-safety property, for some \( \mathcal{R} \).

By Def. 8, safety is a coinductive property [28]: fix \( s \) and \( \mathcal{R} \), \( (s; \mathcal{R}) \)-safe is the largest \( (s; \mathcal{R}) \)-safety property, i.e. the union of all \( (s; \mathcal{R}) \)-safety properties; to prove that some \( \Gamma \) is \( (s; \mathcal{R}) \)-safe, we must find a property \( \varphi \) such that \( \Gamma \in \varphi \), and prove that \( \varphi \) is an \( (s; \mathcal{R}) \)-safety property. Intuitively, we can construct such \( \varphi \) (if it exists) as the set containing \( \Gamma \) and all its reductums (via transition \( \rightarrow \)) and checking whether all elements of \( \varphi \) satisfy all clauses of Def. 8. By clause [\( s \cdot \cdot \cdot \)], whenever two roles \( p \) and \( q \) attempt to communicate, the communication must be possible, i.e. the receiver \( q \) must support all output messages of sender \( p \), with compatible payload types (by rule [\( \Gamma \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \·
crash handling, and any reliable role assumption in the (possibly empty) set \( \mathcal{R} \). The rule can be instantiated by choosing a set \( \mathcal{R} \) and safety property \( \varphi \) (e.g. among the stronger properties presented in Def. 18 later on). Rule \([T-\ell]\) types crashed session endpoints as \texttt{stop}.

The rest of the rules in Fig. 4 are mostly standard. \([T-X]\) looks up process variables. \([T-B]\) types a value \( v \) if it belongs to a basic type \( B \). \([T-Sub]\) holds for a singleton typing context \( c:S \), and applies subtyping when assigning a type \( S' \) to a variable or channel \( c \). \([T-\ell]\) defines a predicate end\((\cdot)\) on typing contexts, indicating all endpoints are terminated – it is used in \([T-0]\) for typing an inactive process \( \mathbf{0} \), and in \([T-\ell]\) for crashed endpoints. \([T-B]\) and \([T-\ell]\) assign selection and branching types to channels used by selection and branching processes. Minor changes w.r.t. standard session types are the clauses “\( S \not\leftrightarrow \text{end} \)” in rules \([T-B]\) and \([T-\text{Call}]\); they forbid sending or passing \texttt{end}-typed channels, while allowing sending/passing channels and data of any other type.\(^1\) Rules \([T-def]\) and \([T-\text{Call}]\) handle recursive processes declarations and calls. \([T-\ell]\) \texttt{linearly} splits the typing context into two, one for typing each sub-process.

### 3.5 Subject Reduction and Session Fidelity

We present our key results on typed processes: \textit{subject reduction} and \textit{session fidelity} (Thms. 11 and 15). A main feature of our theory is that our results explicitly account for the \textit{spectrum} of optional reliability assumptions used during typing.

\[ \text{Subject reduction (Thm. 11 below) states that if a well-typed process } P \text{ reduces to } P', \text{ then the reduction is simulated by its typing context } \Gamma, \text{ provided that the reliability assumptions embedded in } \Gamma \text{ hold when } P \text{ reduces. In other words, if a channel endpoint } s[p] \text{ occurring in } P \text{ is assumed reliable in } \Gamma, \text{ then } P \text{ should } \not\text{ crash } s[p] \text{ while reducing; any other reduction of } P \text{ (including those that crash other session endpoints) are type-safe. To formalise this idea, we define reliable process reduction } \rightarrow_{\Gamma \setminus \mathcal{R}_s}, \text{ as a subset of } P' \text{'s reductions. We also define assumption-abiding reduction } \rightarrow_{\mathcal{R}_s} \text{ to enforce reliable process reductions across nested sessions.} \]

**Definition 10 (Reliable Process Reductions and Assumption-Abiding Reductions).** The reliable process reduction \( \rightarrow_{\Gamma \setminus \mathcal{R}_s} \) is defined as follows:

\[
P \rightarrow P' \quad \forall p \in \mathcal{R}_s : \not\exists R : P' \equiv R | s[p] \not\in \Gamma
\]

Assume \( \Theta \cdot \Gamma \vdash P \) where, for each \( s \in \Gamma \), there is a set of reliable roles \( \mathcal{R}_s \) such that safe\((s; \mathcal{R}_s, \Gamma)\). We define the assumption-abiding reduction \( \rightarrow_{\mathcal{R}_s} \) such that \( P \rightarrow_{\mathcal{R}_s} P' \) holds when: (1) \( P \rightarrow_{\Gamma \setminus \mathcal{R}_s} P'' \) for all \( s \in \Gamma \); and (2) if \( P \equiv (\nu s'), Q \) (for some \( s', \Gamma_s, Q \)) and \( P' \equiv (\nu s') Q' \) and \( Q \rightarrow Q' \), then \( \exists \mathcal{R}_s' \) such that safe\((s'; \mathcal{R}_s', \Gamma_s)\) and \( Q \rightarrow_{\Gamma \setminus \mathcal{R}_s'} Q' \). We write \( \rightarrow_{\mathcal{R}_s} \) for the transitive/reflexive-transitive closure of \( \rightarrow_{\mathcal{R}_s} \).

Hence, when \( P \rightarrow_{\Gamma \setminus \mathcal{R}_s} P' \) holds, none of the session endpoints \( s[p] \) (where \( p \) is a reliable role in set \( \mathcal{R} \)) are crashed in \( P' \). When \( P \) is well-typed, the reduction \( P \rightarrow_{\mathcal{R}_s} P' \) covers all (and

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\(^1\) This restriction is needed for Thms. 11 and 20. It does not limit the expressiveness of our typed calculus, since sending an \texttt{end}-typed channel (not usable for communication) amounts to sending a basic value.
only) the reductions of \( P \) that do not violate any reliability assumption used for deriving \( \Theta \cdot \Gamma \vdash P \); notice that we use congruence \( \equiv \) to quantify over all restricted sessions in \( P \) and ensure their reductions respect all reliability assumptions in their typing, by \([T,T']\) in Fig. 4.

We can now use \( \overset{\rightarrow}{\cdot} \) to state our subject reduction result. Its proof is available in §G.

\[\textbf{Theorem 11 (Subject Reduction).} \text{ Assume } \Theta \cdot \Gamma \vdash P \text{ where } \forall s \in \Gamma : \exists \mathcal{R}_s : \text{safe}(s; \mathcal{R}_s, \Gamma). \text{ If } P \overset{\rightarrow}{\cdot} P' \text{, then } \exists \Gamma' \text{ such that } \Gamma \rightarrow^{s} \chi \Gamma', \text{ and } \forall s \in \Gamma' : \text{safe}(s; \mathcal{R}_s, \Gamma'), \text{ and } \Theta \cdot \Gamma' \vdash P'. \]

\[\textbf{Corollary 12 (Type Safety).} \text{ Assume } \emptyset \cdot \cdot \vdash P \text{. If } P \overset{\rightarrow}{\cdot} P' \text{, then } P' \text{ has no error.} \]

\[\textbf{Example 13 (Subject reduction).} \text{ Take the DNS example (§1) and consider the process acting as the (unreliable) role } q : P_q = \text{safe}(s; \mathcal{R}_q) \oplus s[3] q. \text{ Using type } T_q' \text{ from the same example, can type } P_q \text{ with the typing context } \Gamma_q = s[3] q : T_q'. \text{ Following a crash reduction via } [r,i,r] q, \text{ the process evolves as } P_q \rightarrow P_q' = s[3] q. \text{ Observe that the typing context } \Gamma_q \text{ can reduce to } \Gamma_q = s[p] : \text{stop}, \text{ via } [r,i]; \text{ and by typing rule } [T,i], \text{ we can type } P_q' \text{ with } \Gamma_q'. \]

Session fidelity states the opposite implication w.r.t. subject reduction: if a process \( P \) is typed by \( \Gamma \), and \( \Gamma \) can reduce along session \( s \) (possibly by crashing some endpoint of \( s \)), then \( P \) can reproduce at least one of the reductions of \( \Gamma \) (but maybe not all such reductions, because \( \Gamma \) over-approximates the behaviour of \( P \)). As a consequence, we can infer \( P \)'s behaviour from \( \Gamma \)'s behaviour, as shown in Thm. 20. This result does not hold for all well-typed processes: a well-typed process can loop in a recursion like \( \text{def } X(...) = X \) in \( X \), or deadlock by suitably interleaving its communications across multiple sessions [13]. Thus, similarly to [29] and most session type works, we prove session fidelity for processes with guarded recursion, and implementing a single multiparty session as a parallel composition of one sub-process per role. Session fidelity is given in Thm. 15 below, by leveraging Def. 14.

\[\textbf{Definition 14 (from [29]).} \text{ Assume } \emptyset \cdot \Gamma \vdash P. \text{ We say that } P:\]

(1) has guarded definitions iff in each process definition in \( P \) of the form \( \text{def } X(x_1 : S_1, \ldots, x_n : S_n) = Q \) in \( P' \), for all \( i \in 1..n, \) if \( S_i \) is a session type, then a call \( Y(...) \) can only occur in \( Q \) as a subterm of \( x_i[q] \& \{m_j(y_j) \}.P_j \) or \( x_i[q] \oplus m(d).P'' \) (i.e. after using \( x_i \) for input or output);

(2) only plays role \( p \) in \( s \), by \( \Gamma \) iff: (i) \( P \) has guarded definitions; (ii) \( \forall v(P) = 0; \) (iii) \( \Gamma = \Gamma_0, s[p] : S \) with \( S \not\in \text{end} \) and \( \text{end}(\Gamma_0); \) (iv) for all subterms \( (\nu s' : T')P' \) in \( P, \text{end}(T') \).

We say “\( P \) only plays role \( p \) in \( s \)” iff \( \exists \Gamma : \emptyset \cdot \Gamma \vdash P, \) and item 2 holds.

Item 1 of Def. 14 formalises guarded recursion for processes. Item 2 identifies a process that plays exactly one role on one session; clearly, an ensemble of such processes cannot deadlock by waiting for each other on multiple sessions. All our examples satisfy Def. 14(2).

We can now formalise our session fidelity result (Thm. 15). The statement is superficially similar to Thm. 5.4 in [29], but it now includes explicit reliability assumptions for \( \Gamma \); it also covers more cases, since our typing contexts and processes can reduce by crashing, handling crashes, or losing messages sent to crashed session endpoints. The proof is available in §H.

\[\textbf{Theorem 15 (Session Fidelity).} \text{ Assume } \emptyset \cdot \Gamma \vdash P, \text{ with } \text{safe}(s; \mathcal{R}, \Gamma), \text{ P} = \Pi_{p \in I} P_p, \text{ and } \Gamma = \bigcup_{p \in I} \Gamma_p \text{ such that for each } P_p : (1) \emptyset \cdot \Gamma_p \vdash P_p, \text{ and (2) either } P_p \equiv 0, \text{ or } P_p \text{ only plays } p \text{ in } s, \text{ by } \Gamma_p. \text{ Then, } \Gamma \vdash \forall s.\mathcal{R} \text{ implies } \exists \Gamma', P' \text{ such that } \Gamma \rightarrow s \chi \Gamma', \text{ and } P \overset{\rightarrow}{\cdot} P' \text{, with } \text{safe}(s; \mathcal{R}, \Gamma'), \text{ P'} = \Pi_{p \in I} P'_p, \text{ and } \Gamma' = \bigcup_{p \in I} \Gamma'_p \text{ such that for each } P'_p : (1) \emptyset \cdot \Gamma'_p \vdash P'_p, \text{ and (2) either } P'_p \equiv 0, \text{ or } P'_p \text{ only plays } p \text{ in } s, \text{ by } \Gamma'_p. \]
3.6 Statically Verifying Run-Time Properties of Processes with Crashes

We conclude this section by showing how to infer run-time process properties from typing contexts, even in the presence of arbitrary process crashes. The formulations are based on [29, Def. 5.1 & Fig. 5(1)], but (1) we cater for optional assumptions on reliable roles; (2) a successfully-terminated process or typing context may include crashed session endpoints and failover types/processes (like DNS server \( r \) in §1) that only run after detecting a crash; and (3) non-failover reliable roles terminate by reaching \( 0 \) (in processes) or end (in types).

Def. 16 formalises several desirable process properties, using the assumption-abiding reduction \( \rightsquigarrow \) (Def. 10) to embed any assumptions on reliable roles used for typing. The properties are mostly self-explanatory: deadlock-freedom means that if a process cannot reduce, then it only contains inactive or crashed sub-processes, or recovery processes attempting to detect others' crashes; liveliness means that if a process is trying to perform an input or output, then it eventually succeeds (unless it is only attempting to detect others' crashes).

**Definition 16** (Runtime Process Properties). Assume \( \emptyset \cdot \Gamma \vdash P \) where, \( \forall s \in \Gamma \), there is a set of roles \( \mathcal{R}_s \) such that safe(\( s ; \mathcal{R}_s , \Gamma \)). We say \( P \) is:

1. **deadlock-free** iff \( P \not\rightsquigarrow P' \) implies \( P' \equiv 0 \mid \Pi_{j \in J} s_j[p_j] \mid \Pi_{j \in J} (\text{def } D_{j,1} \text{ in } \ldots \text{def } D_{j,n_j} \text{ in } s_j[p_j][\text{q}_j] \& \text{crash}. Q'_j) \);
2. **terminating** iff it is deadlock-free, and \( \exists j \) finite such that \( \forall n \geq j : P = P_0 \rightsquigarrow P_1 \rightsquigarrow \ldots \rightsquigarrow P_n \) implies \( P_n \not\rightsquigarrow \);
3. **never-terminating** iff \( P \not\rightsquigarrow P' \) implies \( P' \rightarrow \);
4. **live** iff \( P \not\rightsquigarrow P' \equiv C[Q] \) implies:
   - (i) if \( Q = c[q] \oplus m(w). Q' \) then \( \exists C' : P' \rightarrow C'[Q'] \);
   - (ii) if \( Q = c[q] \& \{m_i(x_i).Q'_i\}_{i \in I} \) where \( \{m_i | i \in I\} \neq \{\text{crash}\} \), then \( \exists C', k \in I, w : P' \rightarrow C'[Q'_k(x_{j_k})] \).

In Def. 18 we formalise the type-level properties corresponding to Def. 16. Type-level liveness means that all pending internal/external choices are eventually fired (via a message transmission or crash detection) – assuming fairness (Def. 17, based on strong fairness of components [32, Fact 2]) so all enabled message transmissions are eventually performed.

**Definition 17** (Non-crashing, Fair, Live Paths (adapted from [17, Def. 4.4])). A **non-crashing path** is a possibly infinite sequence of typing contexts \( (\Gamma_n)_{n \in N} \), where \( N = \{0, 1, 2, \ldots\} \) is a set of consecutive natural numbers, and, \( \forall n \in N \), \( \Gamma_n \rightarrow \Gamma_{n+1} \).

We say that a non-crashing path \( (\Gamma_n)_{n \in N} \) is **fair for session** \( s \) iff, \( \forall n \in N \): \( \Gamma_n \overset{s[p]|q\oplus m(S)}{\rightarrow} \), implies \( \exists k, m' \) such that \( N \ni k \geq n \), and \( \Gamma_k \overset{s[p]|m'}{\rightarrow} \Gamma_{k+1} \).

We say that a non-crashing path \( (\Gamma_n)_{n \in N} \) is **live for session** \( s \) iff, \( \forall n \in N \):

1. \( \Gamma_n \overset{s[p]|q\oplus m(S)}{\rightarrow} \), implies \( \exists k, m' \) such that \( N \ni k \geq n \) and \( \Gamma_k \overset{s[p]|m'}{\rightarrow} \Gamma_{k+1} \);
2. \( \Gamma_n \overset{s[q]|p\oplus k(m(N))}{\rightarrow} \), and \( m \neq \text{crash} \) implies \( \exists k, m' \) such that \( N \ni k \geq n \) and \( \Gamma_k \overset{s[p]|m'}{\rightarrow} \Gamma_{k+1} \) or \( \Gamma_k \overset{s[q]|p\oplus k(m(N))}{\rightarrow} \Gamma_{k+1} \).

**Definition 18** (Typing Context Properties). Given a session \( s \) and a set of reliable roles \( \mathcal{R} \), we say \( \Gamma \) is:

1. **(s; \mathcal{R})-deadlock-free** iff \( \Gamma \not\rightsquigarrow \Gamma' \not\rightsquigarrow \not\rightsquigarrow \Gamma' \not\rightsquigarrow \not\rightsquigarrow \not\rightsquigarrow \), implies \( \forall s[p] \in \Gamma : \Gamma(s[p]) \leq \text{end} \) or \( \Gamma(s[p]) = \text{stop} \) or \( \exists q: \Gamma(s[p]) \leq q \& \text{crash}. \Gamma(T) \);
2. **(s; \mathcal{R})-terminating** iff it is deadlock-free, and \( \exists j \) finite such that \( \forall n \geq j : \Gamma = \Gamma_0 \rightarrow \Gamma_1 \rightarrow \Gamma_2 \rightarrow \cdots \rightarrow \Gamma_n \) implies \( \Gamma_n \not\rightsquigarrow \).
\(\phi::=\) (resp. infinite) number of times, where

\[\langle \text{Def. 18} \rangle \text{ and show how they are inherited by typed processes (Thm. 20). In this section,}\]

\(\phi\in\{\text{terminating, never-terminating, live}\}\]

\(\Gamma\) implies \(\Gamma'\) which are fair for session \(s\) are also live for \(s\).

\textbf{Example 19.} Reliability assumptions \(\mathcal{R}\) can affect typing context properties, e.g. consider:

\[\Gamma = s[p]\mu p q@\text{ok}\cdot t_p, s[q]\mu q p@\{\text{ok}, t_q, \text{crash}, \mu t_q r@\{\text{ok}, t_q, \text{crash}\}\}\}, s[r]\mu r q@\text{ok}\cdot t_r\]

If \(\mathcal{R} = \emptyset\), \(\Gamma\) is safe and deadlock-free but not live: if \(p\) does not crash, \(r\)'s \(\text{ok}\) message is never received by \(q\). If we have \(\mathcal{R} = \{r\}\), \(\Gamma\) satisfies never-termination. Here, neither liveness nor termination can be satisfied by adding reliability assumptions. More examples in §C, Ex. 24.

We conclude by showing how the type-level properties in Def. 18 allow us to infer the corresponding process properties in Def. 16. The proof is available in §H.

\textbf{Theorem 20 (Verification of Process Properties).} Assume \(\emptyset\cdot \Gamma \vdash P\), where \(\Gamma\) is \(\langle s; \mathcal{R} \rangle\)-safe, \(P \equiv \Pi_{p \in P^s} P_p\), and \(\Gamma = \bigcup_{p \in P^s} \Gamma_p\) such that for each \(P_p\), we have \(\emptyset\cdot \Gamma_p \vdash P_p\). Further, assume that each \(P_p\) is either \(0\) (up to \(\equiv\)), or only plays \(p\) in \(s\), by \(\Gamma_p\). Then, for all \(\varphi \in \{\text{deadlock-free, terminating, never-terminating, live}\}\), if \(\Gamma\) is \(\langle s; \mathcal{R} \rangle\)-\(\varphi\), then \(P\) is \(\varphi\).

\section{Verifying Type-Level Properties via Model Checking}

In our generalised typing system, we prove subject reduction when a typing context satisfies a safety property (Def. 8); we then give examples of more refined typing context properties (Def. 18) and show how they are inherited by typed processes (Thm. 20). In this section, we highlight a major benefit of our theory: we show how such typing context behavioural properties can be verified using model checkers. We use our typing contexts and their semantics (including crashes and crash handling) as models, and we express our behavioural properties as modal \(\mu\)-calculus formulae; we then use a model checker (mCRL2 [3]) to verify whether a typing context enjoys a desired property.

\textbf{Contexts as Models.} We encode our typing contexts as mCRL2 processes, with LTS semantics that match Def. 7. To embed our optional reliability assumptions, the context encoding reflects the transition relation \(\rightarrow_{\mathcal{R}}\), so it never crashes any reliable role in \(\mathcal{R}\).

\textbf{Properties as Formulae.} A modal \(\mu\)-calculus formula \(\phi\) accepts or rejects a typing context \(\Gamma\) depending on the transition labels \(\Gamma\) can fire while reducing. We write \(\Gamma \models \phi\) when a typing context \(\Gamma\) satisfies \(\phi\). Actions \(\alpha\) range over transition labels in Def. 6; \(d\) (for data) ranges over sessions, roles, message labels, and types. Our formulae \(\phi\) follow a standard syntax:

\[\phi::=\top|\bot|\langle\alpha\rangle\phi|\phi_1\land\phi_2|\phi_1\lor\phi_2|\phi_1\Rightarrow\phi_2|\mu Z.\phi|\nu Z.\phi|Z|\forall d.\phi|\exists d.\phi\]

Truth (\(\top\)) accepts any \(\Gamma\); falsity (\(\bot\)) accepts no \(\Gamma\). The box (resp. diamond) modality, \(\langle\alpha\rangle\phi\) (resp. \(\langle\alpha\rangle\phi\)), requires that \(\phi\) is satisfied in all cases (resp. some cases) after action \(\alpha\) is fired. The least (resp. greatest) fixed point \(\mu Z.\phi\) (resp. \(\nu Z.\phi\)) allows one to iterate \(\phi\) for a finite (resp. infinite) number of times, where \(Z\) denotes a variable for iteration. Lastly, the forms \(\phi_1\Rightarrow\phi_2, \forall d.\phi, \text{ and } \exists d.\phi\) denote implication, and universal and existential quantification.

In Fig. 5 we show the \(\mu\)-calculus formulae corresponding to our properties inDefs. 8 and 18. Compared to [29], such properties are more complex, since they cater for crashes and crash handling transitions. Recall Def. 8, and take a safety property \(\varphi\): for \(\varphi(\Gamma)\) to hold, clause \(s\rightarrow_\mathcal{R}\) requires that whenever \(\Gamma\) can transition to some \(\Gamma'\) (via \(\rightarrow_{\mathcal{R}}\)), then \(\varphi(\Gamma')\) also holds. To represent this clause in modal \(\mu\)-calculus, we use fixed points for possibly infinite paths;
Generalised Multiparty Session Types with Crash-Stop Failures (Tech Report)

14

 Prototype tool that extends To verify the properties in Fig. 5, we implement a Tool Implementation and Example. They interact (sub-formula s)

 Deadlock-freedom, except that it uses the further reduce via s

 action, and either an output or s

 or crash detection action, s

 The first implication requires that, if Γ can fire an s[p]stop action and an input action s[q]p&m(S'), then Γ must be capable of firing a crash handling action, s[q]p.

 Deadlock-Freedom ([µ-safe]) requires that, if Γ is unable to reduce further without crashing (via →), then Γ can hold only ended or stopped endpoints. The antecedent of ⇒ characterises a context that is unable to reduce (since → only allows for transmissions s[p]q&m and crash detection s[p]q); the consequent forbids the presence of any input s[p]q&m(S) or output s[p]q&m(S) transitions. By Def. 7, this means all session endpoints in Γ are ended or stopped.

 Terminating ([µ-term]) holds when Γ can reach a terminal configuration (i.e. cannot further reduce via →) within a finite number of steps. Hence, the formula is similar to deadlock-freedom, except that it uses the least fixed point (µZ,...) to ensure finiteness.

 Never-Terminating ([µ-nterm]) requires that Γ can always keep reducing via → transitions. Therefore, we require some transmission s[p]q&m or crash detection action s[p]q to be always fireable, even after some of the non-reliable roles crash.

 Liveness ([µ-live]) requires that any enabled input/output action is triggered by a corresponding message transmission or crash detection, within a finite number of steps. For input actions (sub-formula φin): if an input s[q]p&m(S) is enabled (left of ⇒), then, in a finite number of steps (µZ,...) involving other roles p', q', a transmission s[p]q&m or a crash detection s[q]p can be fired. For output actions, the sub-formula φout is similar. The µ-calculus formula embeds fairness (Def. 17) by finding some roles p', q' that, no matter how they interact (sub-formula φ′), lead to the desired transmission or crash detection.

 Tool Implementation and Example. To verify the properties in Fig. 5, we implement a prototype tool that extends mpstk [30] (based on the mCRL2 model checker [3]) with support

 Figure 5 Modal µ-Calculus Formulate corresponding to Properties in Defs. 8 and 18, where φ→(Z) = ∀s, p, q, φ→(s, p, q, Z), and φ→(s, p, q, Z) = ∀m. s[p]q&m(Z) & s[p]q&m(Z).

 in Fig. 5 we write φ→(Z) for following a fixed point Z via any transmission, crash,$^2$ or crash handling actions, and we define it as φ→(Z) = ∀s, p, q, m. s[p]q&m[Z] & s[p]q&m[Z].

 Safety ([µ-safe]) requires (in its second implication) that whenever Γ can fire an input action, and either an output or s[q]p&m action, then Γ can also fire a message transmission, s[p]q&m. The first implication requires that, if Γ can fire an s[p]stop action and an input action s[q]p&m(S'), then Γ must be capable of firing a crash handling action, s[q]p.

 Deadlock-Freedom ([µ-def]) requires that, if Γ is unable to reduce further without crashing (via →), then Γ can hold only ended or stopped endpoints. The antecedent of ⇒ characterises a context that is unable to reduce (since → only allows for transmissions s[p]q&m and crash detection s[p]q); the consequent forbids the presence of any input s[p]q&m(S) or output s[p]q&m(S) transitions. By Def. 7, this means all session endpoints in Γ are ended or stopped.

 Terminating ([µ-term]) holds when Γ can reach a terminal configuration (i.e. cannot further reduce via →) within a finite number of steps. Hence, the formula is similar to deadlock-freedom, except that it uses the least fixed point (µZ,...) to ensure finiteness.

 Never-Terminating ([µ-nterm]) requires that Γ can always keep reducing via → transitions. Therefore, we require some transmission s[p]q&m or crash detection action s[p]q to be always fireable, even after some of the non-reliable roles crash.

 Liveness ([µ-live]) requires that any enabled input/output action is triggered by a corresponding message transmission or crash detection, within a finite number of steps. For input actions (sub-formula φin): if an input s[q]p&m(S) is enabled (left of ⇒), then, in a finite number of steps (µZ,...) involving other roles p', q', a transmission s[p]q&m or a crash detection s[q]p can be fired. For output actions, the sub-formula φout is similar. The µ-calculus formula embeds fairness (Def. 17) by finding some roles p', q' that, no matter how they interact (sub-formula φ'), lead to the desired transmission or crash detection.

 Tool Implementation and Example. To verify the properties in Fig. 5, we implement a prototype tool that extends mpstk [30] (based on the mCRL2 model checker [3]) with support

 $^2$ Since our typing contexts encoded in mCRL2 produce ¬γ1×R-transitions that never crash reliable roles in R, our µ-calculus formucle can follow all crash transitions; hence, the formula do not depend on R.
for our crash-stop semantics. The updated tool is available at:

https://github.com/alcestes/mpstk-crash-stop

We now illustrate how this new tool helps in writing correct session protocols with crash handling, and briefly discuss its performance.

In the two-buyers protocol from MPST literature [19], buyers \( b_1 \) and \( b_2 \) agree on splitting the cost of buying a book from seller \( s \). We tackle this protocol with crashes and no reliability assumptions: all roles may crash, and survivors must end the session correctly. The resulting crash-tolerant two-buyers protocol (Fig. 6) is much more complex than the one in the literature. In fact, the possibility of crashes introduces a variety of scenarios where different roles may crash (or not), hence the protocol needs many crash branches. The protocol exhibits two crash-handling patterns: i) exiting gracefully, and ii) recovery behaviour. The former occurs either when \( s \) crashes or when \( b_1 \) crashes prior to the agreed split. The latter occurs should \( b_2 \) crash after the agreed split, whereupon \( b_2 \) concludes the transaction if both \( b_2 \) and \( s \) do not crash. This behaviour is activated via a recovery type in \( s[b_1] \), where the labels \( rp_1 \) and \( rp_2 \) correspond to before the sending of \( addr \); and \( rp_3 \) prior to receiving the \( date \). Overlooking or mishandling some cases is easy; our tool spots such errors, so the protocol can be tweaked until all desired properties hold. We used our tool to verify the protocol: it has 1409 states and 10248 transitions; it is safe, deadlock-free, live, and it is terminating; it is not never-terminating. All properties verify within 100ms on a 4.20 GHz Intel Core i7-7700K CPU with 16 GB RAM. More experimental results can be found in §D.

5 Related Work, Conclusions, and Future Work

Previous Work on Failure Handling in Session Types can be generally classified under two main approaches: affine and coordinator model. The former adapts session types to allow session endpoints to cease prematurely (e.g. by throwing an exception); the latter assumes reliable process coordination to handle failures.

Affine failure handling is first proposed in [24] for a \( \pi \)-calculus with binary sessions (i.e. two roles), and [14] presents a concurrent \( \lambda \)-calculus with binary sessions and exception handling; exceptions are also found in [6, 7]. These works model failures at the application level, via throw/catch constructs. Our key innovations are: (1) we model arbitrary failures (e.g. hardware failures); (2) we specify what to do when a failure is detected at the type level; (3) we support multiparty sessions; and (4) we seamlessly support handling the crash of a role while handling another role’s crash, whereas the do-catch constructs cannot be nested.

Coordinator model approaches include [1], which extends MPST with optional blocks where default values are used when communications fail; and [11], which uses synchronisation

![Figure 6] Two-Buyers protocol extended with crash-handling.
points to detect and handle failures. Both need processes to coordinate to handle failures. [33] extends MPST with a try-handle construct: a reliable coordinator detects and broadcasts failures, and the remaining processes proceed with failure handling. Unlike these works, we do not assume reliable processes, failure broadcasts, or coordination/synchronisation points.

Other papers address failures with different approaches. The recent work [26] annotates global and local types to specify which interactions may fail, and how (process crash, message loss). Their failure model is different from ours; and unlike us, they handle failures by continuing the protocol via default branches and values. Instead, our types include crash branches defining recovery behaviours that are only executed upon crash detection; further, by nesting such crash branches, we can specify different behaviours depending on which roles have crashed. [25] uses an MPST specification to build a dependency graph among running processes, supervise them, and restart them in case of failure. [34] utilises MPST to specify fault-tolerant, event-driven distributed systems, where processes are monitored and restarted if they fail; unlike our work, they require certain reliable roles, but their model tolerates false crash suspicions. More on the theory side, [5] presents a Curry-Howard interpretation of a language with binary session types and internal non-determinism, which is used to model failures (that are propagated to all relevant sessions, similarly to [14,24]). Process calculi with localities have been proposed to model distributed systems with failures [2,8,27]; unlike our work, they do not have a typing system to verify failure handling.

Generalised Multiparty Session Type Systems (introduced in [29]) depart from “classic” MPST [20] by not requiring top-down syntactic notions of protocol correctness (global types, projection, etc.); rather, they check behavioural predicates (safety, liveness, etc.) against (local) session types. [18] adopts the approach to model actor systems with explicit connections in their types [21]. By adopting this general framework, we support protocols not representable as global types in classic MPST (e.g. DNS in §1, two-buyers in §4, and all examples in §D, excepting Adder).

Model Checking Behavioural Types. [9] develops a behavioural type system for the π-calculus, and check LTL formulæ against such types. In [22], the type system combines typing and local analyses, with liveness properties verified via model checking. A similar approach is introduced in [29] for MPST. Regarding applications, [15,23] verify behavioural types extracted from Go source code; and in [31], the Effpi Scala library assigns behavioural types to communicating programs. These works use a model checker to validate e.g. liveness through type-level properties, but do not support crashes or crash handling.

Conclusions and Future Work. We presented a multiparty session typing system for verifying processes with crash-stop failures. We model crashes and crash handling in a session π-calculus and its typing contexts, and prove type safety, protocol conformance, deadlock freedom and liveness. Our system is generalised in two ways: (1) it supports optional reliability assumptions, ranging from fully reliable (as in classic MPST), to fully unreliable (every process may crash); and (2) it is parametric on a behavioural property ϕ (validated by model checking) which can ensure deadlock-freedom, liveness, etc. even in presence of crashes. We also present a prototype implementation of our approach. As future work, we plan to study more crash models (e.g. crash-recover) and types of failure (e.g. link failures). We also plan to study the use of asynchronous global types for specifying protocols with failure handling — but unlike [26], we plan to support the type-level specification of dedicated recovery behaviours that are only executed upon crash detection.
References


A Structural Congruence

The structural congruence relation of our MPST π-calculus, mentioned in Fig. 2, is formalised below. These rules are standard, and taken from [29]; the only extension is rule [C-CrashElim]. Here, fpv(D) is the set of free process variables in D, and dpv(D) is the set of declared process variables in D.

\[
P | Q \equiv Q | P \quad [C-Par] \\
(P | Q) | R \equiv P | (Q | R) \quad [C-Assoc] \\
P | 0 \equiv P \quad [C-ParId]
\]

\[
(\nu s) 0 \equiv 0 \quad [C-ResVar] \\
(\nu s) (\nu s') P \equiv (\nu s') (\nu s) P \quad [C-CrashElim] \\
\]

\[
\text{def } D \equiv 0 \quad [C-DefLift] \\
\text{def } D \in (\nu s) P \equiv (\nu s)(\text{def } D \in P) \quad [C-DefElim]
\]

\[
\text{def } D \equiv 0 \quad [C-DefLift] \\
\text{def } D \in (\nu s) P \equiv (\nu s)(\text{def } D \in P) \quad [C-DefElim]
\]

\[
\text{def } D \equiv 0 \quad [C-DefLift] \\
\text{def } D \in (\nu s) P \equiv (\nu s)(\text{def } D \in P) \quad [C-DefElim]
\]

B Session Subtyping

We formalise our subtyping relation \( \ll \) in Def. 21 below. The relation is mostly standard [29, Def. 2.5], except for the new rule [Sub-stop], and the new (highlighted) side condition “\( |I| = 1 \implies \ldots \)” in rule [Sub-crash]; this condition prevents the supertype from adding input branches to “pure” crash recovery external choices.

\[
\text{Definition 21 (Subtyping). Given a standard subtyping } <: \text{ for basic types (e.g. including int } <: \text{ real), the session subtyping relation } \ll \text{ is coinductively defined:}
\]

\[
B <: B' \quad \text{[Sub-B]} \\
\text{end } \ll \text{ end } \quad \text{[Sub-end]} \\
\forall i \in I \quad S_i \ll S_i' \quad T_i \ll T_i' \quad \text{[Sub-sub]} \\
\text{stop } \ll \text{ stop } \quad \text{[Sub-stop]} \\
\]

\[
\text{Rule [Sub-B] lifts } \ll \text{ to basic types. The rest of the rules say that a subtype describes a more permissive session protocol w.r.t. its supertype. By rule [Sub-sub], the subtype of an internal choice allows for selecting from a wider set of message labels, and sending more generic payloads. By rule [Sub-crash], the subtype of an external choice can support a smaller set of input message labels, and less generic payloads; the side condition “\( |I| = 1 \ldots \)” ensures that if the subtype only has a singleton crash branch, then the same applies to the supertype — hence, both subtype and supertype describe a “pure” crash recovery behaviour, and do not expect to receive any other input.}^3 \text{ By rules [Sub-end] and [Sub-stop], the types end and stop are only subtypes of themselves. Finally, rules [Sub-cr-l] and [Sub-cr-r] say that recursive types are related up to their unfolding. We study the properties of session subtyping in §E.}
\]

\[
^3 \text{ Notice, however, that rule [Sub-crash] allows a supertype to have a crash-handling branch even when the subtype does not have one.}
\]
C Additional Examples

Example 22. We show an example of our crashing semantics. Processes $P$ and $Q$ below communicate on a session $s$; $P$ uses the endpoint $s[p]$ to send an endpoint $s[r]$ to role $q$; $Q$ uses the endpoint $s[q]$ to receive an endpoint $x$, then sends a message to role $p$ via $x$.

$$P = s[p][q] \oplus m'(s[r]), s[p][r] & \& m(x) \quad Q = s[q][p] & \& m'(x), x[p] \oplus m(42)$$

On a successful reduction (without crashes), we have:

\[
\begin{align*}
(vs) \quad (P \mid Q) &= (vs) (s[p][q] \oplus m'(s[r]), s[p][r] & \& m(x) | s[q][p] & \& m'(x), x[p] \oplus m(42)) \\
&\rightarrow (vs) (s[p][r] & \& m(x) | s[p][p] \oplus m(42)) \\
&\rightarrow 0
\end{align*}
\]

Now, suppose that $P$ crashes before sending; this gives rise to the reduction:

\[
\begin{align*}
(vs) \quad (P \mid Q) &= (vs) (s[p][q] \oplus m'(s[r]), s[p][r] & \& m(x) | s[q][p] & \& m'(x), x[p] \oplus m(42)) \\
&\rightarrow (vs) (s[p][q] \oplus m'(s[r]), s[p][r] & \& m(x) | s[q][p] & \& m'(x), x[p] \oplus m(42)) \\
&\rightarrow 0
\end{align*}
\]

We can observe that when the sending process $P$ crashes (by $\mathsf{crash}$, all endpoints in $P$ (i.e. both $s[p]$ and $s[r]$) crash. If $Q$ has a crash handling branch, it can be triggered via $\mathsf{crash}$, suppose instead we have

$$Q' = s[q][p] & \& \{m'(x), x[p] \oplus m(42), \mathsf{crash.0}\}$$

A crash handling reduction can trigger when $P$ crashes:

\[
\begin{align*}
(vs) \quad (P \mid Q') &= (vs) (s[p][q] \oplus m'(s[r]), s[p][r] & \& m(x) | s[q][p] & \& m'(x), x[p] \oplus m(42), \mathsf{crash.0}) \\
&\rightarrow (vs) (s[p][q] \oplus m'(s[r]), s[p][r] & \& m(x) | s[q][p] & \& m'(x), x[p] \oplus m(42), \mathsf{crash.0}) \\
&\rightarrow (vs) (s[p][q] \oplus m'(s[r]), s[p][r] & \& m(x) | s[p][p] \oplus m(42)) \\
&\rightarrow (vs) (s[p][q] \oplus m'(s[r]), s[p][r] & \& m(x) | s[p][p] \oplus m(42)) \\
&\rightarrow 0
\end{align*}
\]

Example 23. Recall the types of the DNS example in §1:

$$T'_p = q \oplus \mathsf{req.} q \& \{\mathsf{res.\, end}\} \quad T'_q = p \& \mathsf{req.} p \oplus \mathsf{res.\, end}$$

Now, consider the following typing context, containing such types:

$$\Gamma = s[p]:T'_p, s[q]:T'_q, s[r]:T'_r$$

Such $\Gamma$ is $(s;\{p, r\})$-safe. We can verify it by checking its reductions. When no crashes occur, we have the following two reductions, where each reductum satisfies Def. 8:

\[
\begin{align*}
\Gamma &\rightarrow_{\mathsf{i}} (s, p) \quad s[p]:q & \& \{\mathsf{res.\, end}\}, s[q]:p \& \mathsf{res}, s[r]:T'_r \\
&\rightarrow_{\mathsf{i}} (s, p) \quad s[p]:\mathsf{end}, s[q]:\mathsf{end}, s[r]:T'_r
\end{align*}
\]

In the case where $q$ crashes immediately, we have:

\[
\begin{align*}
\Gamma &\rightarrow_{\mathsf{i}} (s, p) \quad s[p]:T'_p, s[q]:\mathsf{stop}, s[r]:T'_r \\
&\rightarrow_{\mathsf{i}} (s, p) \quad s[p]:q & \& \{\mathsf{crash.\, res.\, end}\}, s[q]:\mathsf{stop}, s[r]:T'_r \\
&\rightarrow_{\mathsf{i}} (s, p) \quad s[p]:\mathsf{res} \& \mathsf{res}, s[q]:\mathsf{stop}, s[r]:T'_r \\
&\rightarrow_{\mathsf{i}} (s, p) \quad s[p]:\mathsf{res} \& \mathsf{res}, s[q]:\mathsf{stop}, s[r]:\mathsf{p\, res} \\
&\rightarrow_{\mathsf{i}} (s, p) \quad s[p]:\mathsf{end}, s[q]:\mathsf{stop}, s[r]:\mathsf{end}
\end{align*}
\]

and each reductum satisfies Def. 8. The case where $q$ crashes after receiving the $\mathsf{req}$ is similar. There are no other crash reductions to consider, since $p$ and $r$ are reliable.
Example 24. We illustrate safety, deadlock-freedom, liveness, termination, and never-termination over typing contexts via a series of small examples. We first consider the typing context \( \Gamma_A = \Gamma_{Ap}, \Gamma_{Aq}, \Gamma_{Ar} \) where:

\[
\begin{align*}
\Gamma_{Ap} &= s[p] ; \mu t_p.q \oplus \{ok.q \& \{ok.t_p, ko.end, crash.end\}, ko.end\} \\
\Gamma_{Aq} &= s[q] ; \mu t_q.p \& \{ok.p \& \{ok.t_q, ko.end\}, ko.end, crash.r \oplus ok.end\} \\
\Gamma_{Ar} &= s[r] ; p \& \{crash.q \& \{ok.end, crash.end\}\}
\end{align*}
\]

If we assume that all roles in \( \Gamma_A \) are unreliable, \( \Gamma_A \) is safe since its inputs/outputs are dual. However, \( \Gamma_A \) is neither deadlock-free nor live since it is possible for \( p \) to crash immediately before \( q \) sends \( ko \) to \( p \). In such cases, \( q \) will not detect that \( p \) has crashed (since we only detect crashes on receive actions) and terminate without sending a message to the backup process \( r \). This results in a deadlock because \( r \) will detect that \( p \) has crashed, and will expect a message from \( q \).

We observe that changing the reliability assumptions, without changing the typing context, may influence whether a typing context property holds. For example, consider the typing context \( \Gamma_B = \Gamma_{Bp}, \Gamma_{Bq}, \Gamma_{Br} \) where:

\[
\begin{align*}
\Gamma_{Bp} &= s[p] ; \mu t_p.q \oplus ok.t_p \\
\Gamma_{Bq} &= s[q] ; \mu t_q.p \& \{ok.t_q \& \{ok.t_q', crash.end\}\} \\
\Gamma_{Br} &= s[r] ; \mu t_r.q \oplus ok.t_r
\end{align*}
\]

If we assume that all roles are unreliable, \( \Gamma_B \) is safe and deadlock-free but not live — because \( p \) may never crash, and in this case, \( r \)'s outputs are never received by \( q \). Notably, \( \Gamma_B \) is not never-terminating because if both \( p \) and \( r \) crash, then the surviving \( q \) can reach \( end \); however, if we assume that just \( r \) is reliable (i.e. \( \mathcal{R} = \{r\} \)), then \( \Gamma_B \) becomes also never-terminating — because even if both \( p \) and \( q \) crash, role \( r \) can keep running by sending forever \( ok \) messages that are lost (by rule \( r \rightarrow s \) in Fig. 3).

Notice that, in the case of \( \Gamma_B \), we are unable to make liveness hold purely via combinations of reliable roles: this is because (unless \( p \) crashes) \( r \)'s output will never be received by \( q \), irrespective of reliability assumptions. The typing context itself must instead be adapted; for example, by only permitting \( r \) to send once it has detected that \( p \) has crashed.

Instead, in the case of \( \Gamma_A \), we can obtain liveness by adjusting the reliability assumptions: in fact, if we assume \( r \in \mathcal{R} \), then \( \Gamma_A \) is both deadlock-free and live.

Finally, consider the typing context \( \Gamma_C = \Gamma_{ Cp}, \Gamma_{ Cq}, \Gamma_{ Cr} \) where:

\[
\begin{align*}
\Gamma_{ Cp} &= s[p] ; q @ m_1.q \& \{m_2.end, crash.m t_p.r \oplus ok.t_p\} \\
\Gamma_{ Cq} &= s[q] ; p \& \{m_1.p \& m_2.end\} \\
\Gamma_{ Cr} &= s[r] ; p \& \{crash.m t_q.p \& \{ok.t_q\}\}
\end{align*}
\]

\( \Gamma_C \) satisfies safety, deadlock-freedom, and termination when all roles are assumed to be reliable. However, should we instead assume that only \( p \) is reliable, then \( \Gamma_C \) does not satisfy termination. Since external choices in \( \Gamma_C \) do not feature a crash-handling branch when receiving from \( p \), should no roles be assumed reliable, \( \Gamma_C \) satisfies only safety.

D Tool Evaluation

To verify the properties in Fig. 5, we extend the Multiparty Session Types toolKit (mpstk) [30], which uses the mCRL2 model checker [3]. Our extended tool is available at:

https://github.com/alcestes/mpstk-crash-stop

We evaluate our approach with 5 examples: DNS, from §1; Adder, TwoBuyers, and Negotiate, extended from the session type literature [35] with crashes and handling...
The increased model size reflects how the addition of crash handling can complicate even

This form of concludes the sale. It satisfies safety, deadlock-freedom, liveness, and terminating.

\[ s[p, q \rightarrow \text{req}, q \& \{ \text{res.end, crash.r \& req.r \& res.end} \} ] \]
\[ s[p, q \rightarrow \text{req}, p \& \{ \text{res.end} \} ] \]
\[ s[p, q \rightarrow \text{req}, p \& \{ \text{res.end} \} ] \]

\[ s[b1, s \rightarrow (\text{Str}), s \& \{ \text{ok, a} \& \{ \text{Str} \}, s \& \{ \text{Str} \}, s \& \{ \text{Str} \}, s \& \{ \text{Str} \}, T_1 \}, T_2 \} \]
\[ s[b2, s \rightarrow (\text{Str}), s \& \{ \text{ok, a} \& \{ \text{Str} \}, s \& \{ \text{Str} \}, s \& \{ \text{Str} \}, s \& \{ \text{Str} \}, T_1 \}, T_2 \} \]

\[ s[n, c \& \{ o \& \{ \text{Int} \}, p, c \& \{ o \& \{ \text{Int} \}, T_1 \}, c \& \{ o \& \{ \text{Int} \}, T_1 \} \} \]

\[ s[b, n \rightarrow \{ \text{crash} \& \{ o \& \{ \text{Int} \}, p, c \& \{ o \& \{ \text{Int} \}, T_1 \}, T_2 \} \} \]

\[ s[p, q \rightarrow \text{data}(\text{Str}), r \rightarrow \text{data}(\text{Str}), \text{end} \]
Negotiate introduces a (reliable) backup negotiator to the version found in the literature. During normal operation, a client will send an opening offer to a negotiator. Both $c$ and $n$ can then choose to repeatedly exchange counter offers until the other accepts the offer, or rejects it outright, bringing the protocol to an end. In our extension, should the customer detect that the original negotiator crashes, the backup negotiator activates and continues the negotiation with $c$. The example satisfies safety, deadlock-freedom, and liveness. Recovery actions are necessary for $c$ in two locations in order to avoid deadlocks: it is otherwise possible for an offer to be declined or agreed upon, then for $n$ to crash without $c$ noticing; this results in $b$ activating, and expecting a message from the terminated $c$.

Broadcast contains an unreliable broadcaster $p$ attempting to send data to two receivers $q$ and $r$. In cases where $p$ crashes, $r$ requests the data from $q$, who responds with the data it received before $p$ crashed, or with ko when $p$ crashed immediately. The example is not projectable from a global type, since $q$ would otherwise require a message from $r$ even when $p$ had not crashed. Broadcast satisfies safety, deadlock-freedom, liveness, and termination. As in Negotiate, recovery behaviour is necessary for Broadcast to satisfy deadlock-freedom.

Notably, Adder, TwoBuyers and Broadcast have no reliability assumptions: any role may crash at any point. Barring Adder, our examples cannot be written using global types in the session types literature. This demonstrates the flexibility of our generalised MPST system over the classic one. Moreover, the examples include the use of failover processes (DNS and Negotiate) and complex recovery behaviour (TwoBuyers, Negotiate, and Broadcast), thus showcasing the expressivity of our approach.

### D.2 Experimental Results

We applied our extended implementation of mpstk to the examples in Fig. 7. Table 1 gives the full set of verification times, reported in milliseconds with standard deviations, where each time is an average of 30 runs. These results were generated by running mpstk with the

<table>
<thead>
<tr>
<th>$\mathcal{R}$</th>
<th>states</th>
<th>transitions</th>
<th>safe</th>
<th>df</th>
<th>live</th>
<th>nterm</th>
<th>term</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(p,r)$</td>
<td>101</td>
<td>427</td>
<td>12.28 ± 1%</td>
<td>17.14 ± 1%</td>
<td>11.24 ± 1%</td>
<td>15.47 ± 0%</td>
<td>12.33 ± 0%</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>10</td>
<td>15</td>
<td>7.61 ± 1%</td>
<td>8.23 ± 1%</td>
<td>7.46 ± 1%</td>
<td>7.78 ± 1%</td>
<td>7.6 ± 1%</td>
</tr>
<tr>
<td>$(\beta)$</td>
<td>0</td>
<td>37</td>
<td>12.43 ± 0%</td>
<td>15.74 ± 0%</td>
<td>12.24 ± 1%</td>
<td>14.46 ± 0%</td>
<td>12.06 ± 1%</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>26</td>
<td>56</td>
<td>8.92 ± 2%</td>
<td>10.06 ± 0%</td>
<td>8.71 ± 1%</td>
<td>9.42 ± 0%</td>
<td>8.79 ± 0%</td>
</tr>
<tr>
<td>$(\gamma)$</td>
<td>1409</td>
<td>10248</td>
<td>45.6 ± 0%</td>
<td>88.26 ± 0%</td>
<td>31.33 ± 0%</td>
<td>77.2 ± 0%</td>
<td>45.65 ± 0%</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>169</td>
<td>510</td>
<td>11.12 ± 1%</td>
<td>15.94 ± 0%</td>
<td>10.9 ± 1%</td>
<td>12.19 ± 0%</td>
<td>11.06 ± 1%</td>
</tr>
<tr>
<td>$(\delta)$</td>
<td>$b$</td>
<td>1089</td>
<td>34.61 ± 0%</td>
<td>55.07 ± 0%</td>
<td>25.69 ± 0%</td>
<td>47.46 ± 0%</td>
<td>26.04 ± 0%</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>50</td>
<td>157</td>
<td>10.17 ± 0%</td>
<td>12.7 ± 0%</td>
<td>9.93 ± 0%</td>
<td>11.33 ± 0%</td>
<td>9.72 ± 0%</td>
</tr>
<tr>
<td>$(\epsilon)$</td>
<td>0</td>
<td>161</td>
<td>17.99 ± 1%</td>
<td>28.13 ± 0%</td>
<td>14.08 ± 0%</td>
<td>25.72 ± 1%</td>
<td>17.74 ± 0%</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>13</td>
<td>25</td>
<td>7.85 ± 3%</td>
<td>8.65 ± 1%</td>
<td>7.7 ± 0%</td>
<td>8.12 ± 1%</td>
<td>7.85 ± 0%</td>
</tr>
</tbody>
</table>

| Table 1 | Average times (in milliseconds ± std. dev.) for the verification of DNS ($\alpha$), Adder ($\beta$), TwoBuyers ($\gamma$), Negotiate ($\delta$), and Broadcast ($\epsilon$) in Fig. 7 over safety (safe), deadlock-freedom (df), liveness (live), never-terminating (nterm) and terminating (term). Each example has two rows of measurements, varying the sets of reliable roles $\mathcal{R}$: either zero/one/two reliable roles (first row), or all reliable roles (second row). (Benchmarking specs: Intel Core i7-7700K CPU, 4.20 GHz, 16 GB RAM, mCRL2 202106.0 invoked 30 times with: pbes2bool --solve-strategy=2.)
Proof. Similar to [29], except that we have three more cases to consider for the transition \( \Gamma \). For Adder, TwoBuyers, and Negotiate, we use the standard protocol definitions from the literature. For DNS and Broadcast, we omit crash-handling branches. For DNS, this has the consequence of removing the backup role \( r \) entirely.

All examples satisfy safety, deadlock-freedom, and liveness; Adder and Broadcast satisfy termination; no example satisfies never-termination.

Unsurprisingly, all examples demonstrate an increase in verification times and the number of states and transitions when comparing unreliable to reliable versions. Even Adder, which represents minimal crash-handling, demonstrates relevant increases to the number of states and transitions: this is a direct consequence of the unreliable roles, and the resulting generation of crash and crash-detection transitions in the LTS generated by mCRL2. Verification times also increase because the verified properties follow crash and communication actions, thus requiring the exploration of a larger state space compared to the fully reliable versions.

Nevertheless, our verification times do not increase as quickly as the state space grows, and are always under 100 ms. This is because our \( \mu \)-calculus formulas only follow communication, crash, and crash detection transitions, and thus, their verification may not need to follow every possible transition into every state. This suggests greater scalability of the approach that would otherwise be suggested by the size of the state space. This also lends greater motivation to the use of model checking, as it is infeasible to manually determine the properties of a large LTS with complex crash-handling behaviour.

\section{Subtyping Properties}

\begin{lemma}
Assume that \( \Gamma \) is \((s; \mathcal{R})\)-safe and \( \Gamma \leq \Gamma' \Rightarrow \Gamma'' \) with:
\[
\alpha \in \{s[q] ; m, s[q] \oplus r, s[p] ; i \mid q,r \in \mathcal{R}, p \in \mathcal{R}\setminus \mathcal{R}\}
\]

Then, there is \( \Gamma'' \) such that \( \Gamma \Rightarrow \Gamma'' \leq \Gamma'' \).
\end{lemma}

\begin{proof}
Similar to [29], except that we have three more cases to consider for the transition \( \Gamma' \Rightarrow \Gamma'' \).

- \( \Gamma' \xrightarrow{s[p] ; i} \Gamma'' \) with \( p \notin \mathcal{R} \). This means \( \Gamma'(s[p]) \neq \text{stop} \), and thus, \( \Gamma(s[p]) \neq \text{stop} \) (by subtyping); moreover, \( \Gamma'' = \Gamma'[s[p] ; i] \) (by rule \[r;i\] in Def. 7). Therefore, we conclude by taking \( \Gamma'' = \Gamma'[s[p] ; i] \), which implies \( \Gamma'' \leq \Gamma'' \), which is the thesis.

- \( \Gamma' \xrightarrow{s[p] \oplus q} \Gamma'' \). This means \( \Gamma'(s[q]) = \Gamma''(s[q]) = \text{stop} \), and \( \Gamma'(s[p]) = q \land \{m_j(S'_j).T'_j\}_{j \in J} \).

By subtyping, we also have \( \Gamma(s[q]) = \text{stop} \), and \( \Gamma(s[p]) = q \land \{m_i(S_i).T_i\}_{i \in I} \) with \( I \subseteq J \) and \( \forall i \in I : S_i \subseteq S'_i \) and \( T_i \subseteq T'_i \). Since \( \Gamma \) is \((s; \mathcal{R})\)-safe by hypothesis, by clause \( [s;i;k] \) of Def. 8 we know that \( \exists k \in I : m_k = \text{crash} \) — which means that we also have \( \Gamma'' = \Gamma'[T_i/s[p]] \) (since \( \Gamma'[s[p] \oplus q] \), \( \Gamma'' \) and \( k \in I \subseteq J \)). Therefore, we conclude by taking \( \Gamma'' = \Gamma'[T_i/s[q]] \), and we obtain \( \Gamma' \xrightarrow{s[p] \oplus q} \Gamma'' \), which is the thesis.

- \( \Gamma' \xrightarrow{s[p] \ominus q} \Gamma'' \), with \( \Gamma'(s[q]) = \Gamma''(s[q]) = \text{stop} \). This means \( \Gamma'(s[p]) = q \ominus \{m_j(S'_j).T'_j\}_{j \in J} \).

Observe that from \( \Gamma' \xrightarrow{s[p] \ominus q} \Gamma'' \) we have \( \Gamma'' = \Gamma'[T_i/s[p]] \); also observe that since \( k \in J \subseteq I \), we can take \( \Gamma'' \) such that \( \Gamma' \xrightarrow{s[p] \ominus q} \Gamma'' = \Gamma[T_i/s[q]] \), thus also getting \( \Gamma'' \leq \Gamma'' \), this is the thesis.

\end{proof}
Proposition 26. Assume that \( \Gamma \) is \((s; R)-safe\) and \( \Gamma \leq \Gamma' \overset{\alpha_1}{\to} \cdots \overset{\alpha_n}{\to} \Gamma'' \), with:
\[
\forall i \in 1..n : \alpha_i \in \{ s[q]x[r], s[q]s[r], s[p]q \mid q,r \in R, p \in R \setminus R \}
\]
Then, there is \( \Gamma''' \) such that \( \Gamma' \overset{\alpha_1}{\to} \cdots \overset{\alpha_n}{\to} \Gamma''' \leq \Gamma'' \).

Proof. By induction on the number of transitions \( n \) in \( \Gamma' \overset{\alpha_1}{\to} \cdots \overset{\alpha_n}{\to} \Gamma'' \). The base case \( n = 0 \) transitions is immediate: we have \( \Gamma' = \Gamma'' \), hence we conclude by taking \( \Gamma''' = \Gamma \). In the inductive case with \( n = m + 1 \) transitions, there is \( \Gamma_0''' \) such that \( \Gamma' \overset{\alpha_1}{\to} \cdots \overset{\alpha_m}{\to} \Gamma_0''' \overset{\alpha_{m+1}}{\to} \Gamma'' \).

By the induction hypothesis, there is \( \Gamma_0''' \) such that \( \Gamma_0''' \overset{\alpha_1}{\to} \cdots \overset{\alpha_m}{\to} \Gamma_0'' \leq \Gamma_0'' \). Hence, by Lemma 25, there exists \( \Gamma''' \) such that \( \Gamma_0''' \overset{\alpha_1}{\to} \cdots \overset{\alpha_m}{\to} \Gamma''' \) and \( \Gamma''' \leq \Gamma'' \). Therefore, we have \( \Gamma' \overset{\alpha_1}{\to} \cdots \overset{\alpha_n}{\to} \Gamma''' \leq \Gamma'' \), which is the thesis.

Lemma 27. If \( \Gamma \) is \((s; R)-safe\) and \( \Gamma \leq \Gamma' \), then \( \Gamma' \) is \((s; R)-safe\).

Proof. Assume that \( \Gamma \) is \((s; R)-safe\). By contradiction, also assume that \( \Gamma' \) is \( not \) \((s; R)-safe\).

This means that there is a series of reductions \( \Gamma' \overset{\alpha_1}{\to} \cdots \overset{\alpha_n}{\to} \Gamma'' \), with:
\[
\forall i \in 1..n : \alpha_i \in \{ s[q]x[r], s[q]s[r], s[p]q \mid q,r \in R, p \in R \setminus R \}
\]
and with \( \Gamma'' \) violating clause \([S@\&]\) or \([S@\&\&]\) of Def. 8. Now, observe that by Prop. 26, \( \Gamma \) can simulate all such reductions of \( \Gamma' \), reaching a typing context \( \Gamma''' \leq \Gamma'' \); by cases on the subtyping, we can easily verify that \( \Gamma''' \) violates clause \([S@\&\&]\) or \([S@\&\&]\), similarly to \( \Gamma'' \). But then, we obtain that \( \Gamma \) is \( not \) safe either — contradiction. Therefore, we conclude that \( \Gamma' \) is safe.

F Type System Properties

Lemma 28 (Narrowing). If \( \Theta \cdot \Gamma \vdash P \) and \( \Gamma \leq \Gamma' \), then \( \Theta \cdot \Gamma' \vdash P \).

Proof. By induction on the derivation of \( \Theta \cdot \Gamma \vdash P \), we obtain a derivation that concludes \( \Theta \cdot \Gamma' \vdash P \) by inserting (possibly vacuous) instances of rule \([T-Sub]\) (Fig. 4).

Lemma 29 (Substitution). Assume \( \Theta \cdot \Gamma, x:S \vdash P \) and \( \Gamma \vdash w:S \), with \( \Gamma, \Gamma' \) defined. Then, \( \Theta \cdot \Gamma, \Gamma' \vdash P[w/x] \).

Proof. Minor adaptation of [12, Lemma 5].

Lemma 30 (Subject Congruence). Assume \( \Theta \cdot \Gamma \vdash P \) and \( P \equiv P' \). Then, \( \Theta \cdot \Gamma \vdash P' \).

Proof. The proof follows as in [29]. For \([\text{C-CrashElmd}]\), which does not appear,
\[
P = (\nu s)(s[p_1]q \cdots s[p_n]q)
\]
\[
P' = 0
\]
By inversion of \([\text{T}\nu]\), we have \( s \notin \Gamma \) and \( \Theta \cdot \Gamma, \Gamma' \vdash s[p_1]q \cdots s[p_n]q \). Then, by inversion of \([\text{T}\nu]\) and \([\text{T}\nu]\), we have \( \text{end}(\Gamma) \) and \( \forall i \in 1..n : \Theta \cdot \Gamma, s[p_i] : \text{stop} \vdash s[p_i]q \). Therefore, by \([\text{T}\nu]\), we conclude \( \Theta \cdot \Gamma \vdash P' \).
Proofs for Subject Reduction and Type Safety

Proposition 31. If $\Theta \cdot \Gamma \vdash P$ and $P \not= s[p] \mid R$ (for all $s, p, R$), then $\forall c \in \text{dom}(\Gamma)$ : $\Gamma(c) \not= \text{stop}$.

Proof. By easy induction on the derivation of $\Theta \cdot \Gamma \vdash P$.

Proposition 32. If $\Theta \cdot \Gamma \vdash P$, then $\text{fc}(P) \subseteq \text{dom}(\Gamma)$ and $\forall s[p] \in \text{dom}(\Gamma) \setminus \text{fc}(P)$ : $\Gamma(s[p]) \not= \text{end}$.

Proof. By easy induction on the derivation of $\Theta \cdot \Gamma \vdash P$.

Proposition 33. Assume $\Theta \cdot \Gamma \vdash P$. Then, for all $s[p] \in \text{fc}(P)$, we have $\Gamma(s[p]) \not= \text{end}$.

Proof. By induction on the typing derivation of $\Theta \cdot \Gamma \vdash P$, using the rules in Fig. 4. We develop the two most interesting case (the others are similar and easier).

Base case [T-CALL]. We have:

$$P = X(d_1, \ldots, d_n)$$

$$\Gamma = \Gamma_0, \Gamma_1, \ldots, \Gamma_n$$

such that

$$\Theta \vdash X : S_1, \ldots, S_n \text{ end}(\Gamma_0) \quad \forall i \in 1..n \quad \Gamma_i \vdash d_i : S_i \quad S_i \not= \text{end} \quad \text{[T-CALL]} \quad (1)$$

Now observe:

$$\text{fc}(P) \subseteq \{d_i \mid i \in 1..n\} \subseteq \bigcup_{i \in 1..n} \text{dom}(\Gamma_i) \quad \text{(by (1) and Prop. 32)} \quad (2)$$

$$\forall i \in 1..n : d_i \in \text{fc}(P) \implies \Gamma_i(d_i) \not= \text{end} \quad \text{(by (1))} \quad (3)$$

$$\forall s[p] \in \text{fc}(P) : \Gamma(s[p]) \not= \text{end} \quad \text{(by (1), (2), and (3))}$$

which is the thesis.

Inductive case [T-@]. We have:

$$P = q \oplus m(d).P'$$

$$\Gamma = \Gamma_0, \Gamma_1, \Gamma_2$$

such that

$$\Gamma_1 \vdash c : q \oplus m(S).T \quad \Gamma_2 \vdash d : S \quad S \not= \text{end} \quad \Theta \cdot \Gamma_0, c : T \vdash P' \quad \text{[T-@]} \quad (4)$$

Now observe:

$$\text{fc}(P) \subseteq \{c, d\} \cup \text{fc}(P') \subseteq \text{dom}(\Gamma_0) \cup \text{dom}(\Gamma_1) \cup \text{dom}(\Gamma_2) \quad \text{(by (4) and Prop. 32)} \quad (5)$$

$$\Gamma_1(c) \not= \text{end} \quad \text{and} \quad (d \in \text{fc}(P) \implies \Gamma_2(d) \not= \text{end}) \quad \text{(by (4))} \quad (6)$$

$$\forall s[p] \in \text{fc}(P') : (\Gamma_0, c : T)(s[p]) \not= \text{end} \quad \text{(by i.h.)} \quad (7)$$

$$\forall s[p] \in \text{fc}(P') \setminus \{c\} : \Gamma_0(s[p]) \not= \text{end} \quad \text{(by (7))} \quad (8)$$

$$\forall s[p] \in \text{fc}(P) : \Gamma(s[p]) \not= \text{end} \quad \text{(by (4), (5), (8), and (6))}$$

which is the thesis.

Theorem 11 (Subject Reduction). Assume $\Theta \cdot \Gamma \vdash P$ where $\forall s \in \Gamma : \exists R_s : \text{safe}(s; R_s, \Gamma)$. If $P \not\Rightarrow P'$, then $\exists\Gamma' \text{ such that } \Gamma \Rightarrow^* \Gamma'$, and $\forall s \in \Gamma' : \text{safe}(s; R_s, \Gamma')$, and $\Theta \cdot \Gamma' \vdash P'$.

Proof. Let us recap the assumptions:

$$\Theta \cdot \Gamma \vdash P \quad \text{(9)}$$

$$\forall s \in \Gamma : \exists R_s : \text{safe}(s; R_s, \Gamma) \quad \text{(10)}$$

$$P \not\Rightarrow P' \quad \text{(11)}$$
The proof proceeds by induction of the derivation of $P \not\rightarrow_\mathcal{C} P'$, and when the reduction holds by rule $[\text{r-ctx}]$, with a further structural induction on the reduction context $C$. Most cases hold by inversion of the typing $\Theta \cdot \Gamma \vdash P$, and by applying the induction hypothesis.

Case $[\text{r-comp}]$:

\[
P = s[p][q & \{m_i(x_i) \cdot P_i\} \in I] \mid s[q][p] \oplus m_k\langle w \rangle, Q
\]

(by inversion of $[\text{r-comp}]$) (12)

\[
P' = P_k\{w/x_k\} \mid Q \quad (k \in I)
\]

\[
\begin{align*}
\Gamma &= \Gamma_k, \Gamma_0 \quad \text{s.t.} \\
\Theta \cdot \Gamma_k &\vdash s[p][q & \{m_i(x_i) \cdot P_i\} \in I] \\
\Theta \cdot \Gamma_0 &\vdash s[q][p] \oplus m_k\langle w \rangle, Q
\end{align*}
\]

(by (12) and inv. of $[\text{T-\langle} \rangle$) (13)

\[
\Gamma_k = \Gamma_0, \Gamma_1 \quad \text{s.t.} \\
\forall i \in I \quad \Theta \cdot \Gamma_0, x_i : S_i, s[p] : T_i \vdash P_k \\
\Theta \cdot \Gamma_k &\vdash s[p][q & \{m_i(x_i) \cdot P_i\} \in I] \quad [\text{T-\&}]
\]

(by (13) and inv. of $[\text{T-\&}]$) (14)

\[
\Gamma_0 = \Gamma_2, \Gamma_3, \Gamma_4 \quad \text{s.t.} \\
\Gamma_2 &\vdash s[q][p] \oplus \{m_k(\bar{S}_k)' \} \\
\Gamma_3 &\vdash w : S_k' \not\in \text{end} \quad \Theta \cdot \Gamma_2, s[q] : T_k' \vdash Q \\
\Theta \cdot \Gamma_0 &\vdash s[q][p] \oplus m_k\langle w \rangle, Q \quad [\text{T-\&}]
\]

(by (15) and inv. of $[\text{T-\&}]$) (15)

Now, notice that:

\[
\begin{align*}
\Gamma &= \Gamma_0, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4 \\
\Gamma_1 &= s[p] : T \quad \text{with} \quad T \subseteq \{q & \{m_i(S_i) \cdot T_i\} \in I\} \\
\Gamma_4 &= s[q] : T' \quad \text{with} \quad T' \subseteq \{q & \{m_k(S_k)' \cdot T_k'\} \}
\end{align*}
\]

(by (14) and Fig. 4, rule $[\text{T-ctx}]$) (17)

\[
\Gamma' \subseteq \Gamma'' = \Gamma_0, \Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4 \quad \text{where} \\
\Gamma_1' &= s[p] : q & \{m_i(S_i) \cdot T_i\} \\
\Gamma_4' &= s[q] : q & \{m_k(S_k)' \cdot T_k'\}
\]

(by (15), (17), (18), and Def. 5) (19)

\[
\forall s \in \Gamma : \text{safe}(s; \mathcal{R}_s, \Gamma'') \quad \text{(by 10, 19 and Lemma 27)} (20)
\]

\[
k \in I \quad \text{and} \quad S_k' \subseteq S_k
\]

(by (19), (20) and Def. 8, clause $[\text{s-comp}]$) (21)

\[
\Gamma'' \rightarrow \Gamma''' = \Gamma_0, s[p] : T_k, \Gamma_2, \Gamma_3, s[q] : T_k' \\
\forall s \in \Gamma : \text{safe}(s; \mathcal{R}_s, \Gamma''')
\]

(by (19), (21) and Def. 7) (22)

We can now use $\Gamma'''$ to type $P'$:

\[
\Theta \cdot \Gamma_0, x_k : S_k, s[p] : T_k \vdash P_k
\]

(by (21), (15) and (14)) (24)

\[
\Gamma_3 \vdash w : S_k \\
\Gamma_0, \Gamma_3, s[p] : T_k \quad \text{defined}
\]

(by (15), (14), and (13)) (26)

\[
\Theta \cdot \Gamma_0, \Gamma_3, s[p] : T_k \vdash P_k\{w/x_k\}
\]

(by (24), (25), (26), and Lemma 29) (27)

\[
\begin{align*}
\Theta \cdot \Gamma_0 &\vdash s[q][p] \oplus m_k\langle w \rangle, Q \\
\Theta \cdot \Gamma_2 &\vdash s[q][p] \oplus m_k\langle w \rangle, Q
\end{align*}
\]

(by (27), (15), (22), (23) and (12)) (28)

We conclude this case by showing that there exists some $\Gamma'$ that satisfies the statement:

\[
\exists \Gamma' : \Gamma \rightarrow \Gamma' \leq \Gamma''
\]

(by (19), (22), and Lemma 25) (29)

\[
\forall s \in \Gamma' : \text{safe}(s; \mathcal{R}_s, \Gamma')
\]

(by (29) and Def. 8, clause $[\text{s-comp}]$) (30)

\[
\Theta \cdot \Gamma' \vdash P'
\]

(by (28), (29), and Lemma 28)
Case $\text{[R-jn]}$: 

$$
P \equiv s[p] \triangleright s[q][p] \oplus m(s'[x]).Q
$$

(by inversion of $\text{[R-jm]}$) (31)

$$
P' \equiv s[p] \triangleright s'[x] \triangleright Q
$$

(by inversion of $\text{[R-jm]}$) (32)

$$
\Theta \cdot \Gamma \vdash P
$$

end($\Gamma_0$)

(by 31 and inv. of $\text{[R-jn]}$) (33)

$$
\Gamma = \Gamma_j, \Gamma_0 \text{ s.t. } \Theta \cdot \Gamma \vdash P
$$

(by 32 and inv. of $\text{[T-f]}$) (34)

$$
\Gamma_0 = \Gamma_1, \Gamma_2, \Gamma_3 \text{ s.t. }
$$

(by 32 and inv. of $\text{[T-o]}$) (35)

$$
\Gamma_3 = s[q]:T'r \text{ with } T'r \leq p \oplus \{m(S).T\}
$$

(by 32, 33, and 34) (36)

$$
\Gamma \leq \Gamma'' \text{ where }
$$

(by 35, 36, and Def. 5) (37)

$$
\Gamma'' = \Gamma_0, s[p]:\text{stop}, \Gamma_1, s[q]:\text{stop}, s[q]:T'r
$$

(by 15 and Fig. 4, rule $\text{[T-Sub]}$) (38)

$$
\forall s \in \Gamma : \text{safe}(s; \mathcal{R}_s, \Gamma)
$$

(by 10, 37 and Lemma 27) (39)

$$
\Gamma_2(s'[x]) \not\leq \text{stop}
$$

(by 34) (where $S$ cannot be stop) and Def. 21 (40)

$$
\Gamma_2 \vdash s'[x] \triangleright \text{stop}
$$

(by 34) (for $\Gamma_2(s'[x]) \not\leq \text{end}$), (39), and Def. 7 rule $\text{[Γ-f]}$ (41)

$$
\Gamma''\vdash s[q][p] \oplus m(s'[x]).Q
$$

(by 37), (40), Def. 7 rule $\text{[Γ-f]}$) (42)

$$
\forall s \in \Gamma : \text{safe}(s; \mathcal{R}_s, \Gamma')
$$

(by 38), (41), (42 and Def. 8, clause $\text{[s→γ_i]}$) (43)

We can now use $\Gamma''$ to type $P'$:

We conclude this case by showing that there exists some $\Gamma'$ that satisfies the statement:

$$
\exists \Gamma' : \Gamma \rightarrow \Gamma' \triangleright \Gamma''
$$

(by 37), (41), (42 and Prop. 26) (44)

$$
\forall s \in \Gamma' : \text{safe}(s; \mathcal{R}_s, \Gamma')
$$

(by 10, 45 and Def. 8, clause $\text{[s→γ_i]}$) (45)

$$
\Theta \cdot \Gamma' \vdash P'
$$

(by 44), (29), and Lemma 28 (46)

Case $\text{[R-jmB]}$:

$$
P \equiv s[p] \triangleright s[q][p] \oplus m(v).Q
$$

(by inversion of $\text{[R-jmB]}$) (47)

$$
P' \equiv s[p] \triangleright Q
$$

(by inversion of $\text{[R-jmB]}$) (48)
The proof is similar to case \([R-\ominus]\) above, but simpler: since a basic value \(v\) is being sent to a crashed endpoint \(s[p]\), we have that \(P'\) does not contain a new crashed session endpoint \(s'[\mathcal{R}]\), and the typing context \(\Gamma_2\) (which types the message payload \(v\)) is empty (by rule \([T-B]\) in Fig. 4). Consequently, we can adapt the proof by omitting the crashed endpoint \(s'[\mathcal{R}]\), skipping step (40), and adjusting step (41) to have \(\Gamma'' \xrightarrow{\{s[p]:P''\}} \Gamma''\). 

Case \([R-\ominus]\):

\[
P = s[p][q] & \{m(x_i).P_i, \text{crash}.P''\}_{i \in I} \mid s[q]
\]

(by inversion of \([R-\ominus]\))

(47)

\[
\begin{align*}
\Theta \cdot \Gamma_k & \vdash s[p][q] & \{m(x_i).P_i, \text{crash}.P''\}_{i \in I} \\
\Theta \cdot \Gamma_j & \vdash s[q] & \{\}
\end{align*}
\]

(by (47) and inv. of \([R-\ominus]\))

(48)

\[
\Gamma = \Gamma_k, \Gamma_j \text{ s.t. } \Theta \cdot \Gamma_k \vdash s[p][q] & \{m(x_i).P_i, \text{crash}.P''\}_{i \in I} \quad \Theta \cdot \Gamma_j \vdash s[q] & \{\}
\]

(by (48) and inv. of \([T-\text{case}]\))

(49)

\[
\Gamma_2 = \Gamma_0, \Gamma_1, s[q]:\text{stop} \text{ s.t. } \Theta \cdot \Gamma_2 \vdash s[q]:\text{stop} \vdash s[q].
\]

(by (48) and inv. of \([T-\text{case}]\))

(50)

Now, notice that:

\[
\Gamma = \Gamma_0, \Gamma_1, \Gamma_2, s[q]:\text{stop}
\]

(by (48), (49), and (50))

(51)

\[
\Gamma_1 = s[p]:T' \quad \text{with} \quad T' \leq q & \{m(S_i).T_i, \text{crash}.T\}_{i \in I}
\]

(by (49) and Fig. 4, rule \([T-\text{send}]\))

(52)

\[
\Gamma \leq \Gamma'' \quad \text{where} \quad \Gamma'' = \Gamma_0, s[p]:q & \{m(S_i).T_i, \text{crash}.T\}_{i \in I}, \Gamma_2, s[q]:\text{stop}
\]

(by (51), (52), and Def. 5)

(53)

\[
\Gamma'' \xrightarrow{s[p]} \Gamma'' = \Gamma_0, s[p]:T, \Gamma_2, s[q]:\text{stop}
\]

(by (53) and Def. 7)

(54)

We can now use \(\Gamma''\) to type \(P'\):

\[
\Theta \cdot \Gamma_0, s[p]:T \vdash P'' \quad \Theta \cdot \Gamma_2, s[q]:\text{stop} \vdash s[q].
\]

(by (49), (50), (54), and (47))

(55)

We conclude this case by showing that there exists some \(\Gamma'\) that satisfies the statement:

\[
\exists \Gamma' : \Gamma \rightarrow \Gamma' \leq \Gamma''
\]

(by (53), (54), and Lemma 25)

(56)

\[
\forall s \in \Gamma' : \text{safe}(s; \mathcal{R}_s, \Gamma')
\]

(by (56) and Def. 8, clause \([S-\text{q}]\))

(57)

\[
\Theta \cdot \Gamma' \vdash P'
\]

(by (55), (29), and Lemma 28)
Case \([R\cdot I\&]:\)

\[
P = s[p][q] \oplus m(w).Q
\]
\[
P' = \Pi_{j \in J} s_j[p_j] : where \{s_j[p_j]\}_{j \in J} = fc(P)
\]

(by inversion of \([R\cdot I\&]) \ (58)

\[
\Gamma_2 \vdash s[p]+q \oplus \{m(S).T\}
\]
\[
\Gamma \vdash w:S
\]

\[
\Gamma = \Gamma_0, \Gamma_1, \Gamma_2 \text{ s.t. } \Theta \cdot \Gamma \vdash s[p]+q \oplus m(w).Q \quad \text{[T-\&]} \quad \text{(by (58), inv. of } [R\cdot I\&]) \ (59)
\]

\[
\forall j \in J : s_j[p_j] \not\in \text{end} \quad \text{(by (58), (59), and Prop. 33) (60)
\]

Now, notice that:

\[
\forall j \in J : \Gamma'(s_j[p_j]) = \text{stop}
\]

(by (60), rule \([T\cdot i]) in Def. 7) \ (61)

\[
\Theta \cdot \Gamma' \vdash P'
\]

(by (58), (61), and \([T\cdot i]) and \([T\cdot \&]) \ (62)

\[
\forall j \in J : p_j \not\in \mathcal{R}_{s_j}
\]

(by (58) and 11) \ (63)

\[
\forall s \in \Gamma' : \text{safe}(s; \mathcal{R}_s, \Gamma')
\]

(by 10, (61), (63) and Def. 8, clause \([s\rightarrow j]]) \ (64)

Hence, we obtain the thesis by (61), (64) and (62).

Case \([R\cdot f\&]): similar to case \([R\cdot f\&]) above, except that we proceed by inversion of \([R\cdot f\&]).

Cases \([R\cdot Ctx] \) and \([R\cdot Ctxf] \). The proofs for these two cases are similar. By inversion of the rule and Def. 2, we have to prove the statement in the following sub-cases:

\(\begin{align*}
\text{[R-Ctx]} & \quad (1) \; P = Q \mid R \text{ and } P' = Q' \mid R \text{ and } Q \rightarrow Q' \\
\text{[R-Ctx]} & \quad (2) \; P = (\nu s')Q \text{ and } P' = (\nu s')Q' \text{ and } Q \rightarrow Q' \\
\text{[R-Ctx]} & \quad (3) \; P = \text{def } D \in Q \text{ and } P' = \text{def } D \in Q' \text{ and } Q \rightarrow Q'
\end{align*}\)

Cases 1 and 3 are easily proved using the induction hypothesis. Therefore, here we focus on case 2.

\[
\Gamma' = \{s'[p]:T_p\}_{p \in I}
\]
\[
\text{safe}(s'; \mathcal{R}', \Gamma')
\]
\[
\text{s' \not\in} \quad \text{by 2 and inv. of } [T\cdot \nu]) \ (65)
\]

\[
\exists \Gamma', \mathcal{R}' \text{ s.t. } \Theta \cdot \Gamma \vdash P \quad \text{[T\cdot \nu]} \quad \text{(by (58), inv. of } [T\cdot \nu]) \ (59)
\]

\[
\begin{align*}
\Gamma'_s = \{s'[p]:T'_p\}_{p \in I} \\
\text{safe}(s'; \mathcal{R}', \Gamma'_s)
\end{align*}
\]
\[
\Gamma' \not\in \mathcal{R}' \quad \text{by (65) and i.h.)} \ (66)
\]

\[
\begin{align*}
\text{Case 2.} & \\
\forall p \in \mathcal{R}' : \exists R : Q' & \equiv R \mid s[p] \not\in \text{stop} \quad \text{(by 2, 11 and Def. 10) (67)}
\end{align*}
\]

\[
\forall p \in \mathcal{R}' : \Gamma'_s(s'[p]) \neq \text{stop} \quad \text{(by 67 and Prop. 31) (68)
\]

\[
\text{safe}(s'; \mathcal{R}', \Gamma'_s) \quad \text{(by (65), (66), (68) and Def. 8, clause } [s\rightarrow j])} \ (69)
\]

\[
\Gamma'_s = \{s'[p]:T'_p\}_{p \in I}
\]
\[
\text{safe}(s'; \mathcal{R}', \Gamma'_s)
\]
\[
\text{s' \not\in} \quad \text{(by (65), (66), (68) and Def. 8, clause } [s\rightarrow j]) \ (69)
\]

\[
\Theta \cdot \Gamma' \vdash P' \quad \text{[T\cdot \nu]} \quad \text{(by (66), (69) and 2) (70)
\]

Hence, we obtain the thesis by (66) and (70).}

\textbf{Corollary 12 \text{(Type Safety).} Assume } \emptyset \cdot \emptyset \vdash P. \text{ If } P \not\rightarrow s P', \text{ then } P' \text{ has no error.}
Proof. From the hypothesis $P \not\rightarrow^* P'$, we know that $P \equiv P_0 \not\rightarrow P_1 \not\rightarrow \ldots \not\rightarrow P_n = P'$ (for some $n$). The proof proceeds by induction on $n$. The base case $n = 0$ is immediate: we have $P = P'$, hence $P'$ is well-typed — and since the term $\text{err}$ is not typeable, $P'$ cannot contain such a term. In the inductive case $n = m + 1$, we know (by the induction hypothesis) that $P_m$ is well-typed, and we apply Thm. 11 to conclude that $P_{m+1} = P'$ is also well-typed and has no $\text{err}$ subterms.

H Proofs for Session Fidelity and Process Properties

\textbf{Theorem 15 (Session Fidelity).} Assume $\emptyset \cdot \Gamma \vdash P$, with $\text{safe}(s; \mathcal{R}, \Gamma)$, $P \equiv \Pi_{p \in I} P_p$, and $\Gamma = \bigcup_{p \in I} \Gamma_p$ such that for each $P_p$: (1) $\emptyset \cdot \Gamma_p \vdash P_p$, and (2) either $P_p \equiv 0$, or $P_p$ only plays $p$ in $s$, by $\Gamma_p$. Then, $\Gamma \not\rightarrow_{i \cdot \mathcal{R}} \vdash \exists \Gamma', P'$ such that $\Gamma \not\rightarrow_{i \cdot \mathcal{R}} \Gamma'$, $P \not\rightarrow_{i \cdot \mathcal{R}} P'$ and $\emptyset \cdot \Gamma' \vdash P'$, with $\text{safe}(s; \mathcal{R}, \Gamma')$, $P' \equiv \Pi_{p \in I} P'_p$, and $\Gamma' = \bigcup_{p \in I} \Gamma'_p$ such that for each $P'_p$: (1) $\emptyset \cdot \Gamma'_p \vdash P'_p$, and (2) either $P'_p \equiv 0$, or $P'_p$ only plays $p$ in $s$, by $\Gamma'_p$.

\textbf{Proof.} The proof structure is similar to Thm. 5.4 in [29]: by induction on the derivation of the reduction of $\Gamma$, we infer the contents of $\Gamma$ and then the shape of $P$ and its sub-processes $P_p$, showing that they can mimic the reduction of $\Gamma$. The main differences w.r.t. [29] are that (1) we now account for crashed session endpoints with type $\text{stop}$; and (2) the proof covers more cases, as it now includes crash detection reductions, and outputs to crashed processes.

Compared to the proof of Thm. 5.4 in [29], we have the following additional cases to consider when a crash is detected, or a selection targets a crashed process.

- case $\Gamma \xrightarrow{s[p]\langle q \rangle} \Gamma'$. In this case, the process $P_p$ playing role $p$ in session $s$ is a branching on $s[p]$ from $q$ (possibly within a process definition) including crash detection; therefore, $P_p$ can correspondingly detect that the channel endpoint $s[q]$ is crashed, by rule $[\text{R-cr}]$ in Fig. 2 (possibly after a finite number of transitions under rule $[\text{R-x}]$). The resulting continuation process $P'$ is typed by $\Gamma'$;

- case $\Gamma \xrightarrow{s[p]\langle q \rangle} \Gamma'$ and $\Gamma(s[q]) = \text{stop}$. In this case, the process $P_p$ playing role $p$ in session $s$ is a selection on $s[p]$ towards $q$ (possibly within a process definition); therefore, $P$ could correspondingly reduce to $P'$ by sending either a basic value $v$ or a channel endpoint $s'[p']$ (possibly after a finite number of transitions under rule $[\text{R-x}]$) to the crashed channel endpoint $s[q]$. We have two possible cases for the communication reduction leading from $P$ to $P'$:

  - rule $[\text{R-\langle \rangle}]$ in Fig. 2, with $s'[p']$ crashed in $P'$. This case is impossible: in fact, by the side condition of the typing rule $[\text{R-cr}]$ (Fig. 4), we must have $\Gamma_p(s'[p']) \not\vdash \text{end}$ — and this would contradict the assumption that $P'_p$ only plays role $p$ in session $s$, by $\Gamma'_p$;

  - rule $[\text{R-\langle p \rangle}]$ in Fig. 2. In this case, we have $\Gamma_p \xrightarrow{s[p]\langle q \equiv \text{err}(B) \rangle} \Gamma'_p$ (for some $m$ and basic type $B$), and the continuation process $P'$ is typed by the resulting $\Gamma'$.

Prop. 34 below says that if a process $P$ satisfies the assumptions of session fidelity (Thm. 15) then all its reducts will satisfy such assumptions, too. This means that if $P$ satisfies session fidelity, then all its reducts enjoy session fidelity, too.

\textbf{Proposition 34.} Assume $\emptyset \cdot \Gamma \vdash P$, where $\Gamma$ is $(s; \mathcal{R})$-safe, $P \equiv \Pi_{p \in I} P_p$, and $\Gamma = \bigcup_{p \in I} \Gamma_p$ such that, for each $P_p$, we have $\emptyset \cdot \Gamma_p \vdash P_p$. Further, assume that each $P_p$ is either $\emptyset$ (up to $\equiv$), or only plays $p$ in $s$, by $\Gamma_p$. Then, $P \not\rightarrow_{i \cdot \mathcal{R}} \vdash \exists \Gamma'$ such that $\Gamma \not\rightarrow_{i \cdot \mathcal{R}} \Gamma'$ and $\emptyset \cdot \Gamma' \vdash P'$, with $\Gamma'$ $(s; \mathcal{R})$-safe, $P' \equiv \Pi_{p \in I} P'_p$, and $\Gamma' = \bigcup_{p \in I} \Gamma'_p$ such that, for each $P'_p$, we have $\emptyset \cdot \Gamma'_p \vdash P'_p$; furthermore, each $P'_p$ is $\emptyset$ (up to $\equiv$), or only plays $p$ in $s$, by $\Gamma'_p$.

\textbf{Proof.} Straightforward from the proof of Thm. 11, which accounts for all possible transitions from $P$ to $P'$, and in all cases yields the desired properties for its typing context $\Gamma'$. 

\hspace*{1cm}
Theorem 20 (Verification of Process Properties). Assume $\emptyset.\Gamma \vdash P$, where $\Gamma$ is $(s; \mathcal{R})$-safe, $P \equiv \Pi_{p \in P}(P_p)$, and $\Gamma = \bigcup_{p \in P} \Gamma_p$ such that for each $P_p$, we have $\emptyset.\Gamma_p \vdash P_p$. Further, assume that each $P_p$ is either $0$ (up to $\equiv$), or only plays $p$ in $s$, by $\Gamma_p$. Then, for all $\varphi \in \{\text{deadlock-free, terminating, never-terminating, live}\}$, if $\Gamma$ is $(s; \mathcal{R})$-safe, then $P$ is $\varphi$.

Proof. Deadlock-freedom Consider any $P'$ such that $P \xrightarrow{\tau} P' \not\rightarrow$ with $P = P_0 \xrightarrow{\tau} P_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} P_n = P' \not\rightarrow$ (for some $n$) with each reduction $P_i \xrightarrow{\tau} P_{i+1}$ ($i \in \{0, \ldots, n-1\}$) satisfying Def. 10. By Prop. 34, we know that each $P_i$ is well-typed and its typing context $\Gamma_i$ is such that $\Gamma \rightarrow^{\exists s \in \mathcal{R}} \Gamma_i$; moreover, $P_i$ satisfies the single-session requirements of Thm. 15. Now observe that, since the process $P_n = P' \not\rightarrow$ cannot reduce further (except by crashing), by the contrapositive of Thm. 15 we obtain $\Gamma_n \not\rightarrow$; and since $\Gamma$ is $(s; \mathcal{R})$-deadlock-free by hypothesis, by Def. 18 (item Deadlock-freedom) we have $\forall s[p] \in \Gamma_n : \Gamma_n(s[p]) \leq \text{end}$ or $\Gamma_n(s[p]) = \text{stop}$ or $\Gamma_n(s[p]) \leq \text{q\&crash}.T'$. Therefore, by inversion of typing, we have $P' \equiv 0 \Pi_{p \in \Gamma} s[p] \not\rightarrow \Pi_{p \in \Gamma} (\text{def} D_{j,1} \ldots \text{def} D_{j,n})$, where $s[p]$ is non-guarded (Def. 16, item Deadlock-freedom) is the thesis.

Terminating We know that $\exists j$ finite such that, $\forall n \geq j$, $\Gamma_0 \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \Gamma_1 \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \ldots \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \Gamma_n$ implies $\Gamma_0 \not\rightarrow$; moreover, since $\Gamma$ is $(s; \mathcal{R})$-deadlock-free (by Def. 18, item 2), whenever $\Gamma_0 \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \Gamma_1 \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \ldots \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \Gamma_n \not\rightarrow$ (for any $n$), then $\forall s[p] \in \Gamma_n : \Gamma_n(s[p]) \leq \text{end}$ or $\Gamma_n(s[p]) = \text{stop}$ or $\Gamma_n(s[p]) \leq \text{q\&crash}.T'^\prime$. Considering all the possible sequences of reductions of $P$, we have the following cases:

1. $P = P_0 \xrightarrow{\tau} P_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} P_m$ and $P_m \not\rightarrow$ (for some $m$). Since $\Gamma$ is $(s; \mathcal{R})$-deadlock-free (by Def. 18, item 2), by item Deadlock-freedom above, $P_m \equiv 0 \Pi_{p \in \Gamma} s[p] \not\rightarrow \Pi_{p \in \Gamma} (\text{def} D_{j,1} \ldots \text{def} D_{j,n})$, where $s[p]$ is non-guarded (Def. 16, item Deadlock-freedom). Now, if we admit it, by the proof of Thm. 11 (we have two possibilities (both leading to a contradiction):

- there is an infinite sequence of typing context reductions $\Gamma = \Gamma_0 \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \Gamma_1 \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \ldots \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \Gamma_n \not\rightarrow$ to type each $P_i$ with a suitable $\Gamma_j$ (with $j \leq i$); moreover, for each such $\Gamma_j$, we have $\Gamma_j \not\rightarrow$ (otherwise, $\Gamma$ would not be $(s; \mathcal{R})$-deadlock-free, hence by Def. 18, item 2, it would also not be $(s; \mathcal{R})$-terminating). But then, we contradict the hypothesis that $\exists j$ finite such that, $\forall n \geq j$, $\Gamma = \Gamma_0 \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \Gamma_1 \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \ldots \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \Gamma_n$ implies $\Gamma_0 \not\rightarrow$;

- there are infinitely many processes reductums that can be typed by a same $\Gamma$. By the proof of Thm. 11, this can only happen in the following ways (all leading to a contradiction):

- firing infinitely many reduction within some restricted session — which would contradict the hypothesis that each parallel sub-process of $P$ only plays one role in session $s$ (Def. 14);

- performing infinitely many process calls by firing rule $\text{[R-X]}$ (in Fig. 2) infinitely many times, without other message transmissions or error detection reductions (which would cause the typing context to reduce). However, this would contradict the hypothesis that $P$ has guarded definitions (Def. 14);

- having a recursive protocol in $\Gamma$ such that $\Gamma_i = \Gamma_{i+1}$. This would lead to the same contradiction addressed in the first case above.

Summing up, all possible sequences of reductions of $P$ are finite, and they are all deadlock-free — which is the thesis.

Never-Terminating By hypothesis and Def. 18 (item 3), we know that $\Gamma \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \Gamma'$ implies $\Gamma \not\rightarrow$. By contradiction, assume that $P$ is never-terminating, i.e. $\exists P'$ such that $P = P_0 \xrightarrow{\tau} P_1 \xrightarrow{\tau} \cdots \xrightarrow{\tau} P_n = P' \not\rightarrow$. By Prop. 34, we know that each $P_i$ is well-typed and its typing context $\Gamma_i$ is such that $\Gamma \rightarrow^{\exists \forall \cdot j \in \mathcal{R}} \Gamma_i$; moreover, $P_i$ satisfies the single-session requirements of Thm. 15. Now observe that, since the process $P_n = P' \not\rightarrow$ cannot reduce further (except by crashing), by the contrapositive of Thm. 15 we obtain $\Gamma' \not\rightarrow$ — but this contradicts the hypothesis that $\Gamma$ is $(s; \mathcal{R})$-never-terminating. Therefore, we conclude that $P$ is never-terminating.
Live  By contradiction, assume that $P$ is not live. Since (by hypothesis) each parallel component of $P$ only plays one role $p$ in session $s$, this means that there are $P', C, Q$ such that $P = P_0 \xrightarrow{\ell} P_1 \xrightarrow{\ell} \ldots \xrightarrow{\ell} P_n = P' \equiv C[Q]$ where either:

- $Q = s[p][q] \oplus m(w).Q'$ (for some $m, w, Q'$), and $\beta C': P' \rightarrow^* C'[Q']$. By Prop. 34, we know that each $P_i$ is well-typed and its typing context $\Gamma_i$ is such that $\Gamma \rightarrow^* \neg s \cdot R \Gamma_i$; moreover, each $P_i$ satisfies the single-session requirements of Thm. 15. Therefore, $P'$ satisfies the single-session requirements of Thm. 15, and is typed by some $\Gamma'$ such that $\Gamma \rightarrow^* \neg s \cdot R \Gamma_i$ — hence, by inversion of typing, $Q$ is typed by some $\Gamma'_p$ (part of $\Gamma'$) where $\Gamma'_p(s[p])$ is a (possibly recursive) internal choice towards $q$, including a choice $m(S)$ (where $S$ types the message payload $w$). Therefore, we have $\Gamma' \xrightarrow{s[p][q] \oplus m(S)}$. Now, recall that (for the sake of the proof by contradiction) we are assuming that no sequence of reductions of $P'$ can fire the top-level selection of $Q$; this means that no parallel component of $P'$ ever exposes an external choice by role $q$ including message label $m$; correspondingly, there is at least one fair and non-crashing path beginning with $\Gamma'$ (yielded by Thm. 11) that never fires a transmission label $s[p][q]m'$ (for any $m'$). But then, such a fair path starting from $\Gamma'$ is not live, hence (by Def. 18, item 4) we obtain that $\Gamma$ is not live — contradiction;

- $Q = s[p][q] \& \{m_i(x_i).Q'_i\}_{i \in I}$ (for some $I, m_i, x_i, Q'_i$ such that either $|I| \neq 1$ or $\forall i \in I: m_i \neq \text{crash}$), and $\beta C', k \in I, w: P' \rightarrow^* C'[Q'_k[w/x_k]]$. The proof is similar to the previous case, and reaches a similar contradiction.

Summing up, we have shown that if we assume $P$ not live, we reach a contradiction. Therefore, we conclude that $P$ is live. \hfill □